Priority Queues

cs151

Priority Queue

- A queue that maintains order of elements according to some priority
- Contrast to Queue which is FiFo

- **PriorityQueues are about the order in which things are removed, NOT the way in which they are stored.**
    - the items may or may not be sorted, or otherwise arranged.
    - This statement applies to stack and queues also, it is just convenient in those cases to arrange data to make retrieval easy

Complexity Analysis

	Unordered	**Ordered (using SAL)**	**Heap Based**
offer	O(1)	O(n)	O(lg n)
peek	O(n)	O(1)	O(1)
poll	O(n)	O(1)	O(lg n)

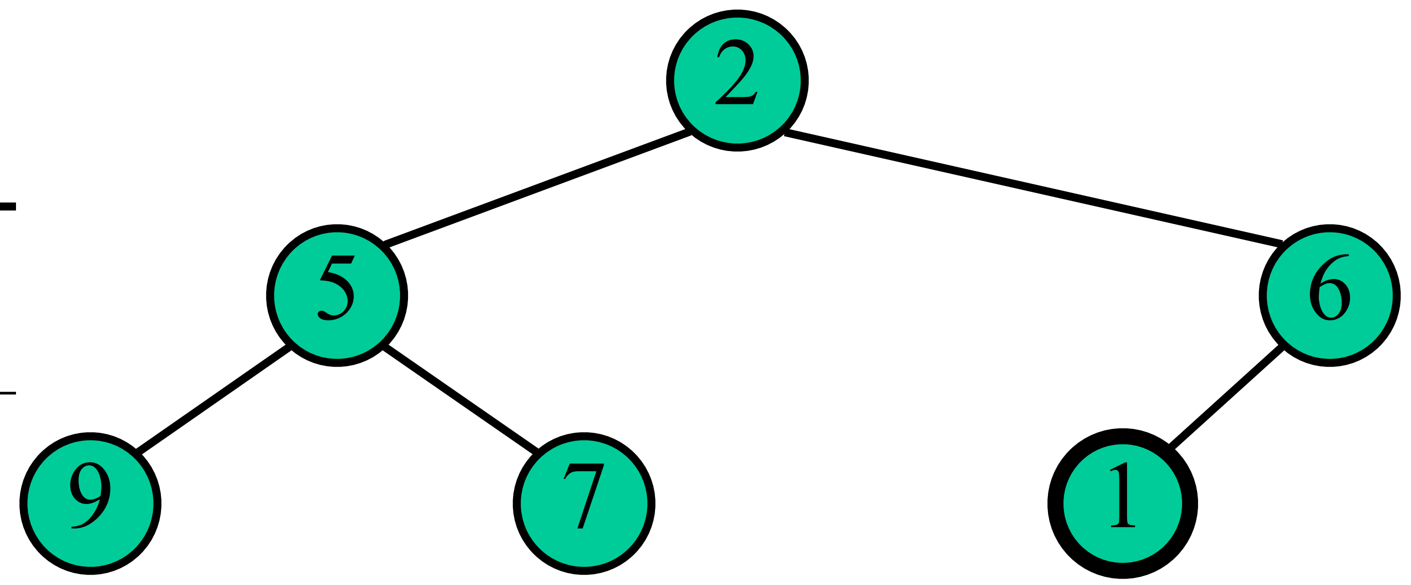
Unordered PQ == Selection Sort

Ordered PQ = Insertion Sort

Binary Heap

- A heap is a “binary tree” storing keys at its nodes and satisfying:
  - heap-order: for every internal node v other than root, $key(v) \geq key(parent(v))$
  - Heap is filled from top down and within a level from left to right.
    - at depth h, the leaf nodes are in the leftmost positions
    - last node of a heap is the rightmost node of max depth

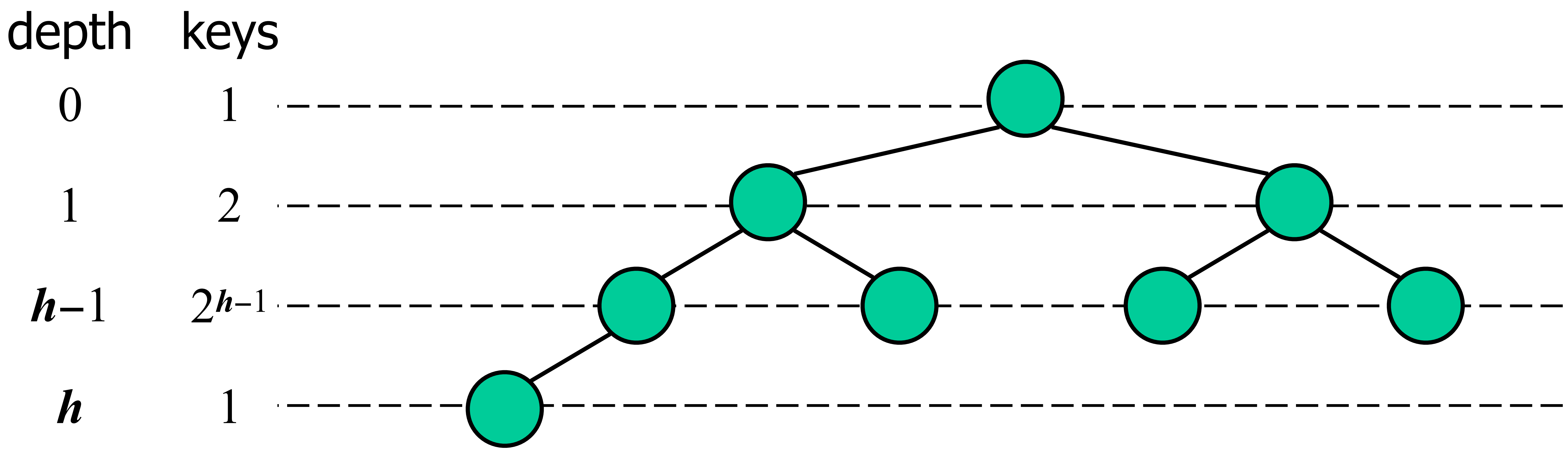
Binary Tree — terms



Term	Definition	
Node	A part of a tree.	
Parent	A node that has children	
Child	A node that has parents. Child nodes have exactly one parent	
Binary Tree	A structure of nodes such that parent nodes have at at most two children	
Root	The node in a tree that has no parent.	
Leaf	Any node that has no children	
Height	The maximum distance from a the root node to a leaf.	
Subtree	The part of a tree whose root is a given node	

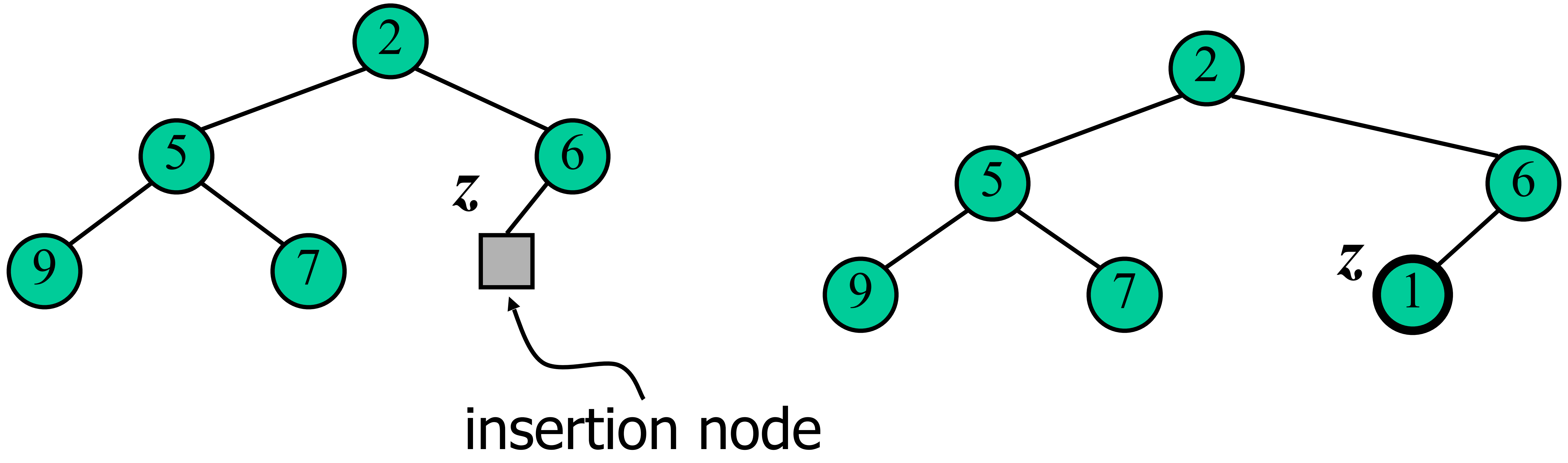
Height of a Heap

- A binary heap storing n keys has a height of $O(\log_2 n)$
- This is NOT true for general binary trees



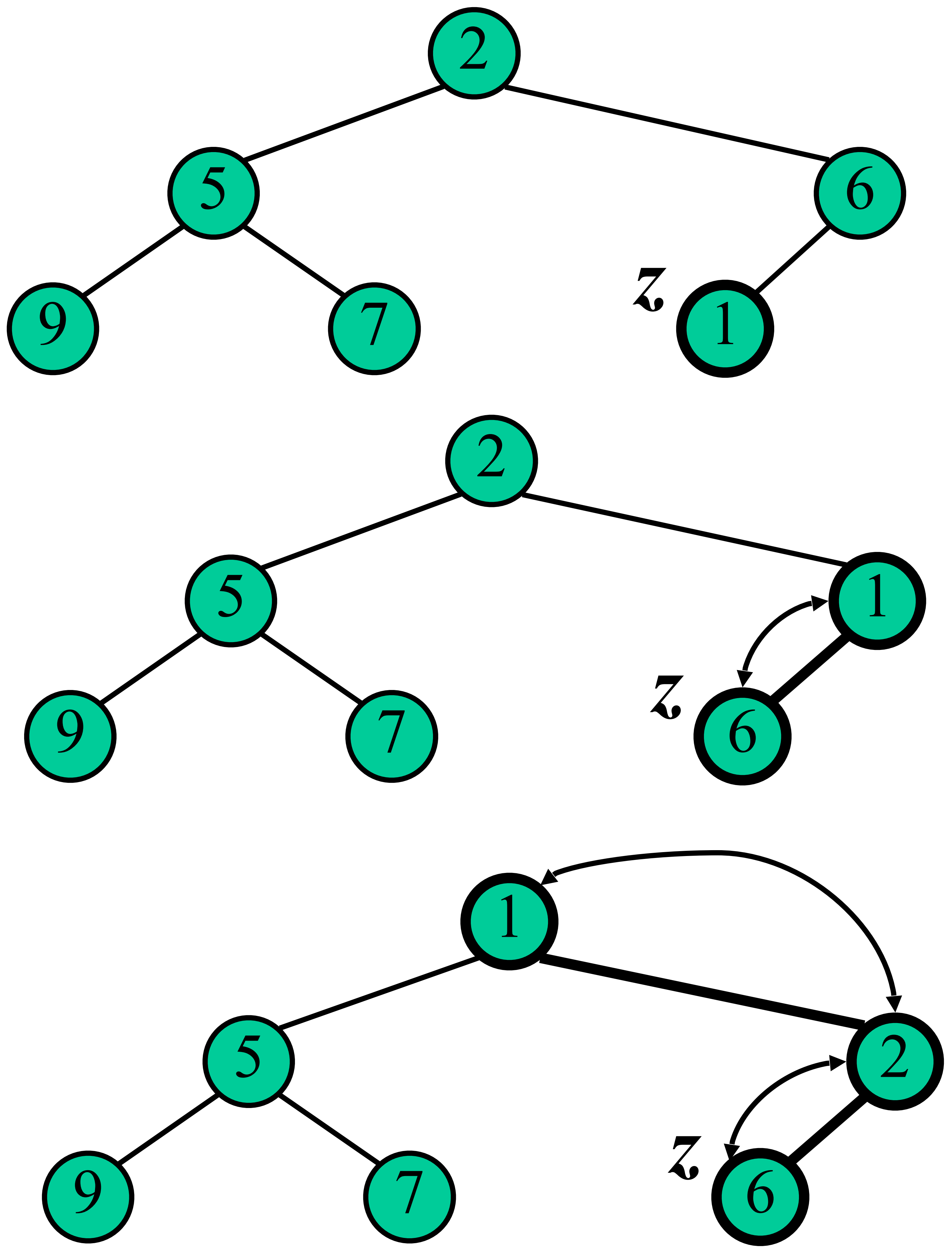
Insertion into a Heap

- Insert as new last node
- Need to restore heap order



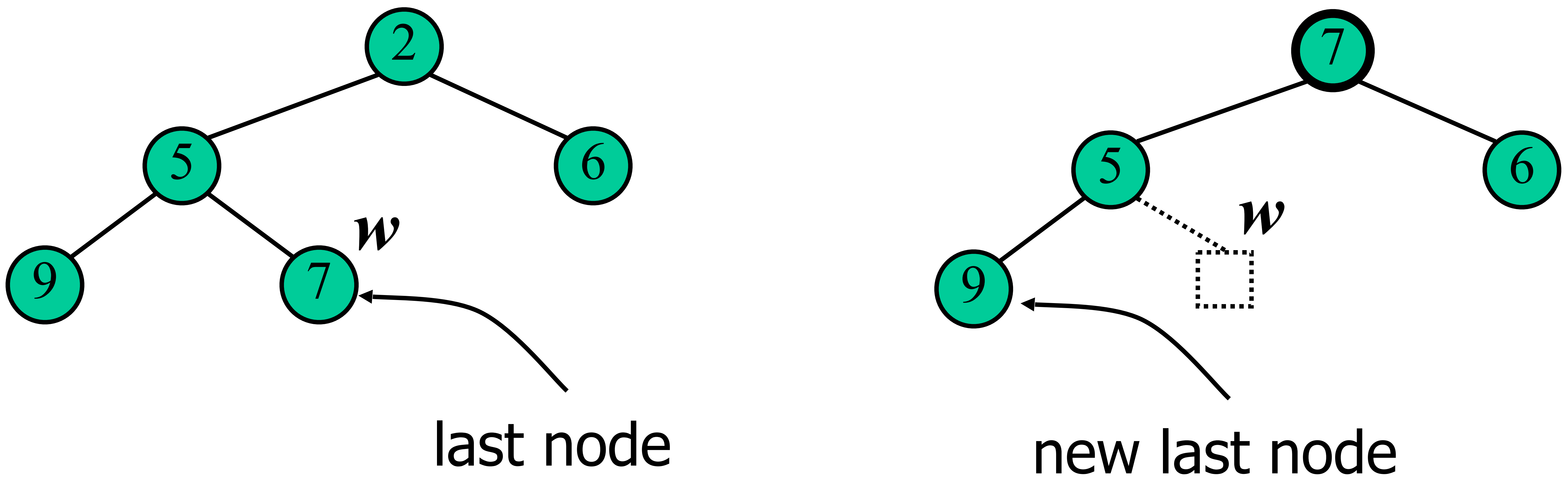
Upheap

- Restore heap order
  - swap upwards
  - stop when finding a smaller parent
  - or reach root
- $O(logn)$



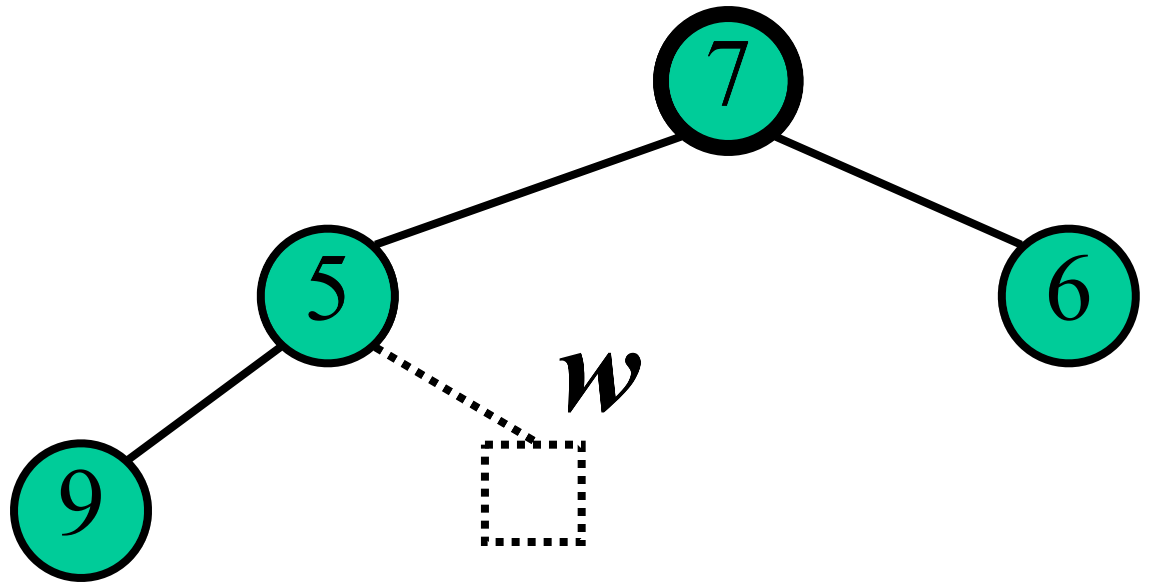
Poll

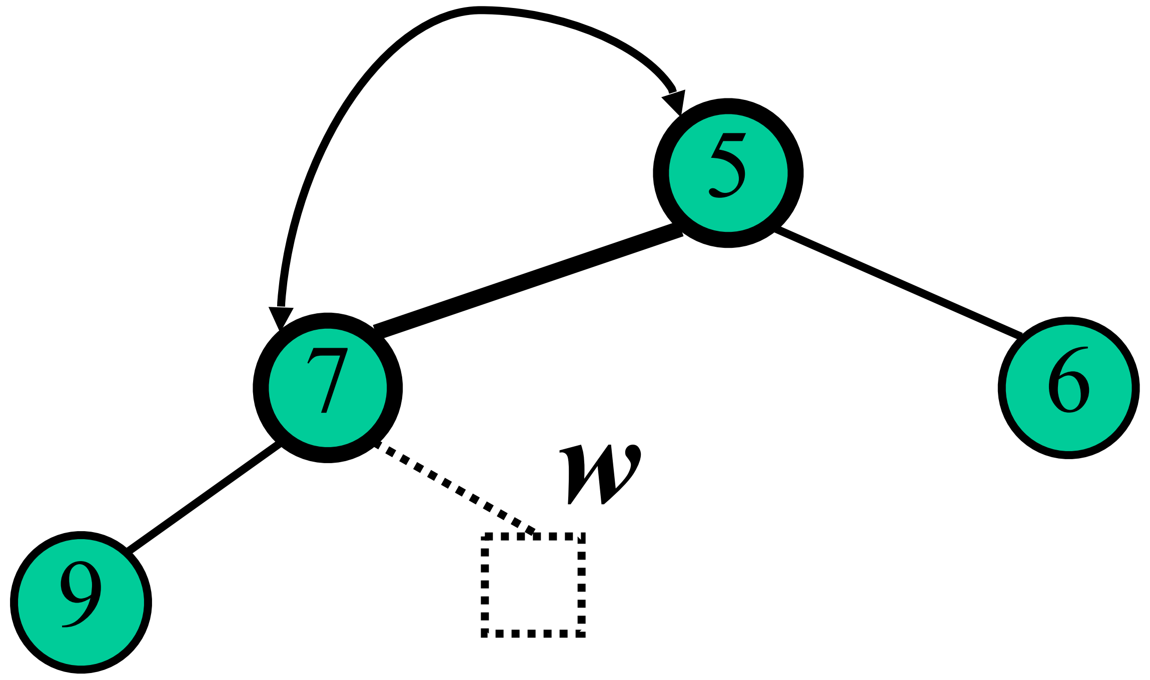
- Removing the root of the heap
  - Replace root with last node
  - Remove last node
  - Restore heap order



Downheap

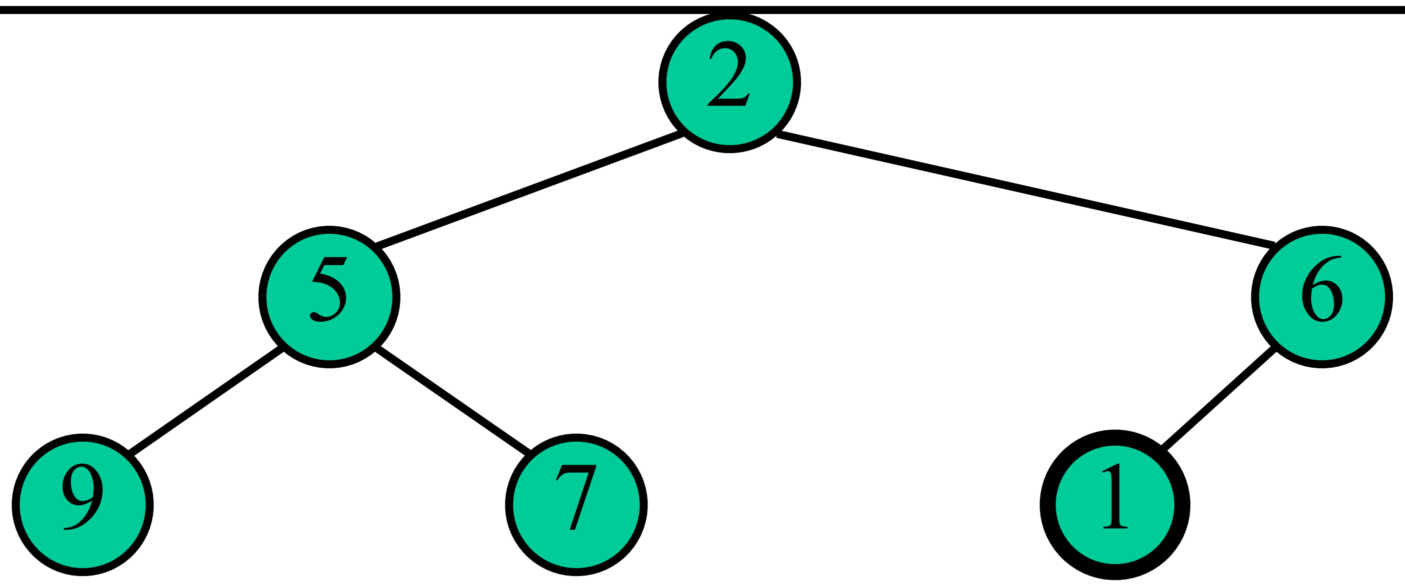
- Restore heap order
  - swap downwards
  - swap with smaller child
  - stop when finding larger children
  - or reach a leaf
- $O(\log n)$





Heaps are built on Arrays

End of Midterm 2



Add to Heap here

0	1	2	3	4	5	6	7
1	5	2	9	7	6		

Locations of Parents and children are in strict mathematical relationship

- Parent from child
  - suppose child is at location childLoc in array
    - parentLoc = (childLoc-1)/2
- Child from Parent
  - suppose parent is at parentLoc in array
    - leftChild = parentLoc*2+1
    - rightChild = parentLoc*2+2

- Parent from child
  - child at loc 4 (value 7)
  - parent is at (4-1)/2 = 1 (value 5)
- Child from Parent
  - parent at loc 2 (value 6)
    - leftChild =2*2+1 = 5 (value 1)
    - rightChild = 2*2+2 = 6 (value — not used)

Priority Queue using Heaps

startup

```
public class PriorityQHeap<K extends Comparable<K>, V> extends AbstractPriorityQueue<K, V>
{

   private static final int CAPACITY = 1032;
   private Pair<K,V>[] backArray;
   private int size;

    public PriorityQHeap() {
        this(CAPACITY);
    }

    public PriorityQHeap(int capacity) {
        size=0;
        backArray = new Pair[capacity];
    }
    @Override
    public int size()
    {
        return size;
    }

    @Override
    public boolean isEmpty()
    {
        return size==0;
    }
```

Heap Insertion

Priority Queue offer method

```
public boolean offer(K key, V value)
```

1. Ensure there is room — if not return false
2. Add new items to end of heap (low and left viewed graphically) first unoccupied viewed array-wise
3. Repeat until at root
    1. Compare with parent
    2. If greater, swap and continue
    3. If less stop
4. return true

Peek and Poll

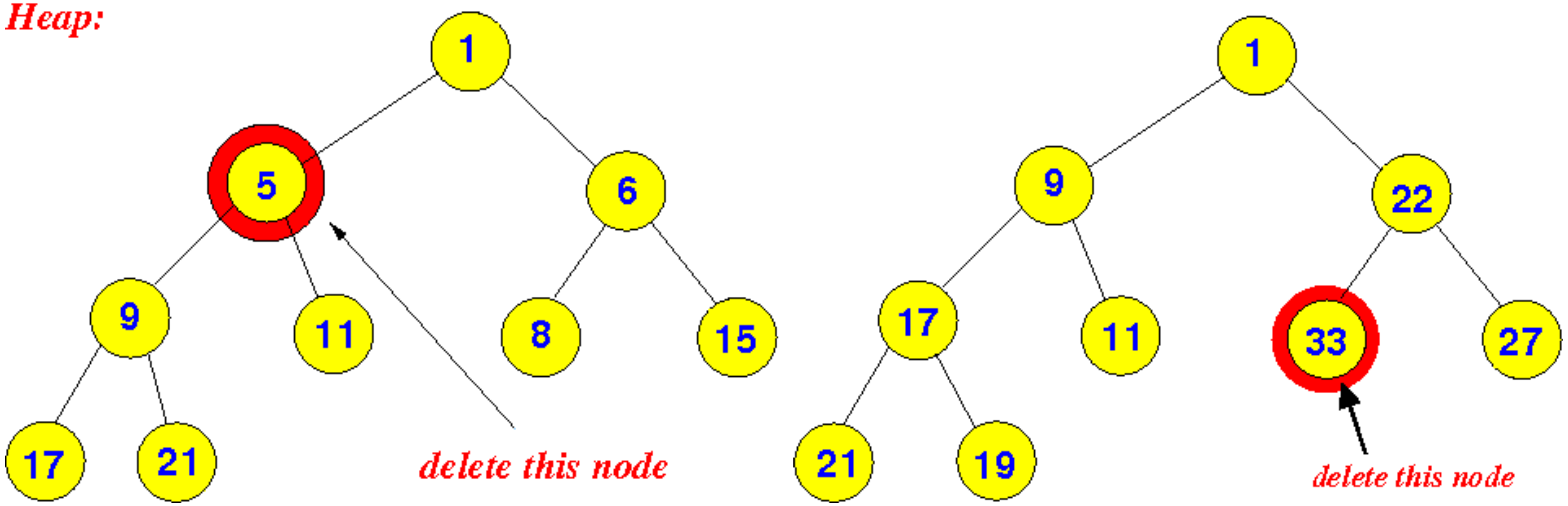
```java
@Override
public V poll() {
    if (isEmpty())
        return null;
    Entry<K,V> tmp = backArray[0];
    removeTop();
    return tmp.theV;
}

@Override
public V peek() {
    if (isEmpty())
        return null;
    return backArray[0].theV;
}
```

Remove head item from Heap

```
private void removeTop()
{
    backArray[0] = backArray[size-1];
    backArray[size-1]=null;
    size--;
    int upp=0;
    while (true)
    {
        int dwn;
        int dwn1 = upp*2+1;
        if (dwn1>size) break;
        int dwn2 = upp*2+2;
        if (dwn2>size) {  dwn=dwn1;
        } else {
            int cmp = backArray[dwn1].compareTo(backArray[dwn2]);
            if (cmp<=0)  dwn=dwn1;
            else dwn=dwn2;
        }
        if (0 > backArray[dwn].compareTo(backArray[upp]))
        {
            Pair<K,V> tmp = backArray[dwn];
            backArray[dwn] = backArray[upp];
            backArray[upp] = tmp;
            upp=dwn;            }
        else {  break;            } } }
```

General Removal

- swap with last node
- delete last node
- may need to upheap or downheap



Heap Insertion

Priority Queue offer method

```
public boolean offer(K key, V value)
    {
        if (size>=(backArray.length-1))
            return false;
    // put new item in at end data items
    int loc = size++;
    backArray[loc] = new Pair<K,V>(key, value);
    // up heap
    int upp = (loc-1)/2; //the location of the parent
    while (loc!=0) {
        if (0 > backArray[loc].compareTo(backArray[upp])) {
            // swap and climb
            Pair<K,V> tmp = backArray[upp];
            backArray[upp] = backArray[loc];
            backArray[loc] = tmp;
            loc = upp;
            upp = (loc-1)/2;
            }
        else
        {
            break;
        }
    }
    return true;
    }
```