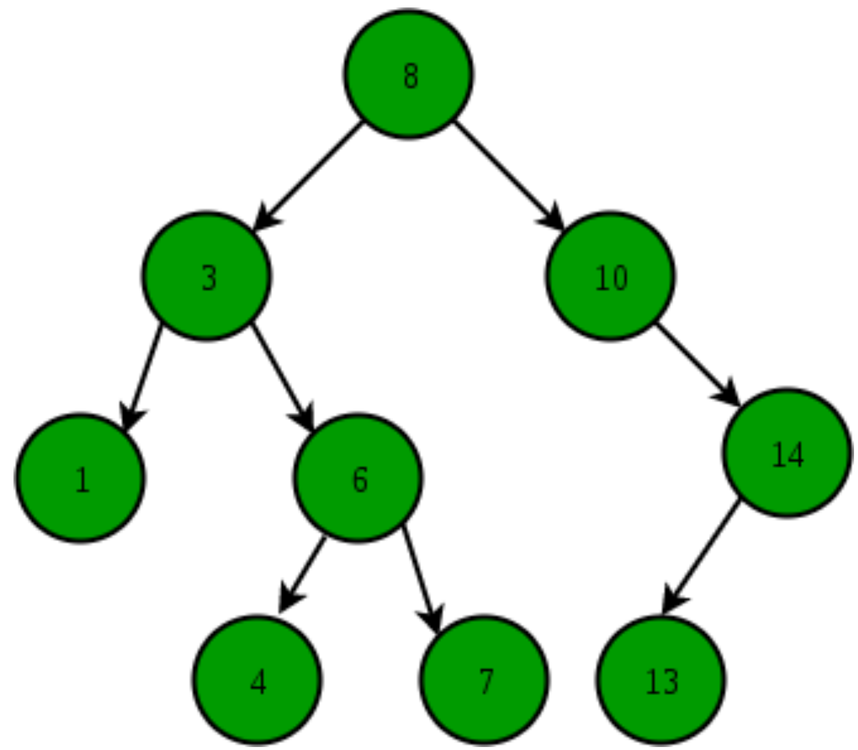
CS206

Search Trees, AVL Trees

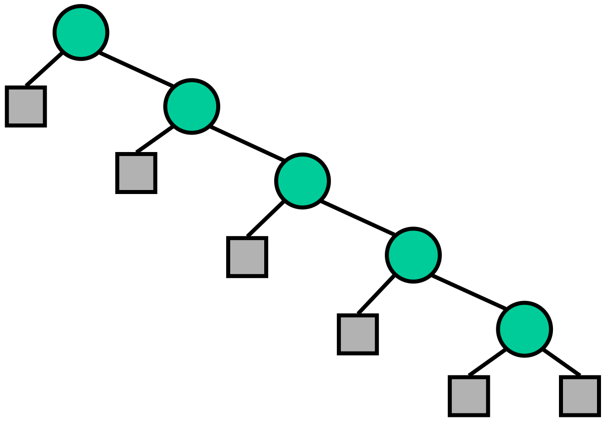
Binary Search Trees

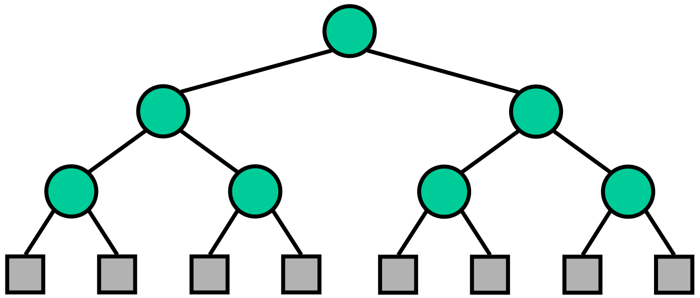
- For all nodes
  - The left node is less than parent
  - The right node is greater than parent



Binary Search Trees

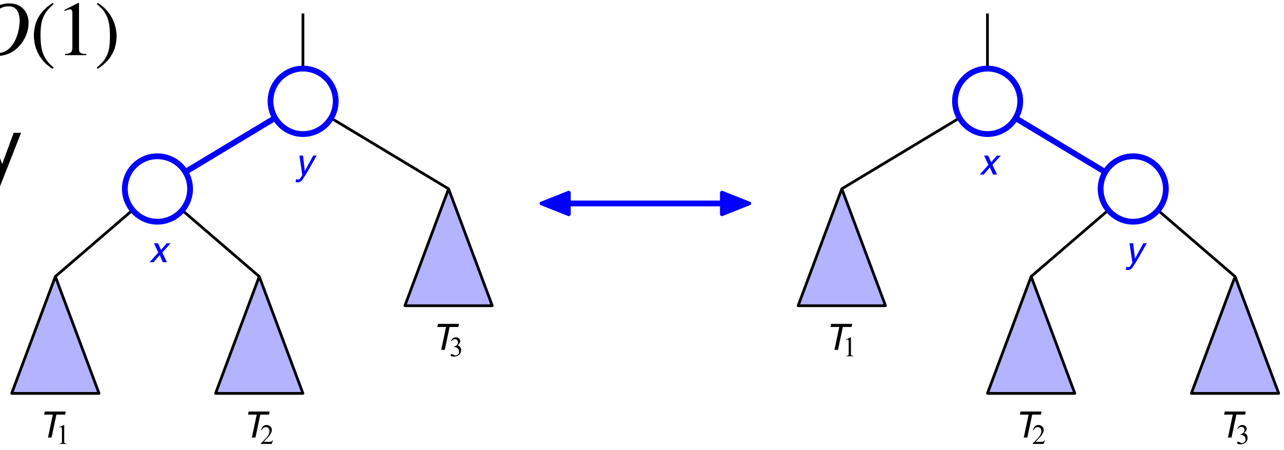
- Performance is directly affected by the height of tree
- All operations are $O(h)$
- $h = O(n)$ worst case
- $h = O(logn)$ best case
- Expected $O(logn)$ if tree is "balanced"
  - balance — generally same number of nodes in left and right subtrees





Balanced Search Trees

- A variety of algorithms augment a standard BST with occasional operations to reshape, reduce height and maintain balance.
- General approach: Rotation — moves a child to be above its parent,
  - ideally $O(1)$
  - certainly O(lgn)



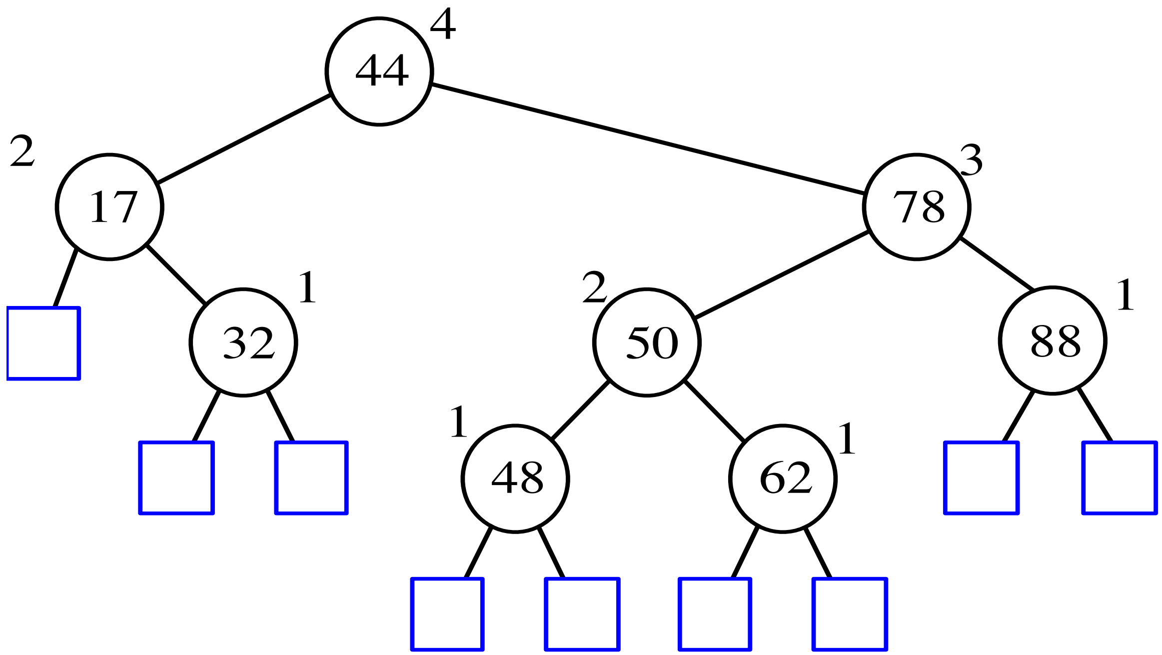
Rotation Algorithms

- **AVL trees**
  - Adelson-Velski and Landis (1962)
- Splay trees
- (2,4) trees
  - non-binary trees
- Red-Black trees

AVL Trees

- Height-balance property
  - For every internal node, the avlHeight of the two children differ by at most 1
    - avlHeight = max distance from null endpoint
- Any binary tree satisfying the height-balance property is an AVL tree
- A height-balanced tree has height O(lg n)
  - max height is provably 1.44*lg(n)

AVL Tree Example



Insertion

- Maintain with each node the avlHeight.
- On insertion, first recur down through tree to insert.
- Then as you unwind recursion, update the avlHeight of each node.
- If height changes, check the height of other child
  - if not in balance then fix

Insertion code to maintain height

(the only code today!!!)

```
private class Node {
    Comparable<E> element;
    int avlHight;
    Node right;
    Node left;

    public Node(Comparable<E> e) {
        avlHight = 1;
        element=e;
        right=null;
        left=null;
    }
}
```

More insertion (pseudo)code

```
int insertUtil(node, element):
    if element==node.payload
        return -1;

    avlD=2;
    if node.payload > element:
      if node.left==null
        node.left=new Node(payload)
      else
        avlD = 1+insertUtil(node.left,element);
    else
        // same but for right

    node.avlHieght = greater of avlD and
                     node.avlHeight

    return node.avlHeight
```

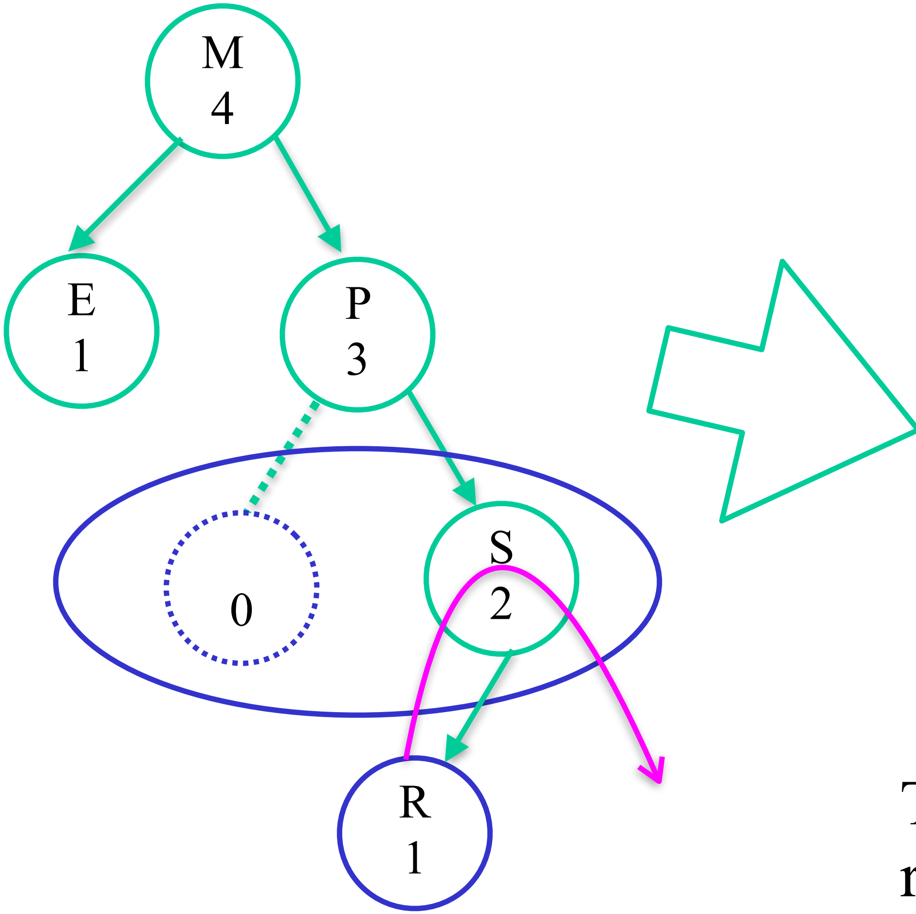
Fixing height imbalances
Rotation!!

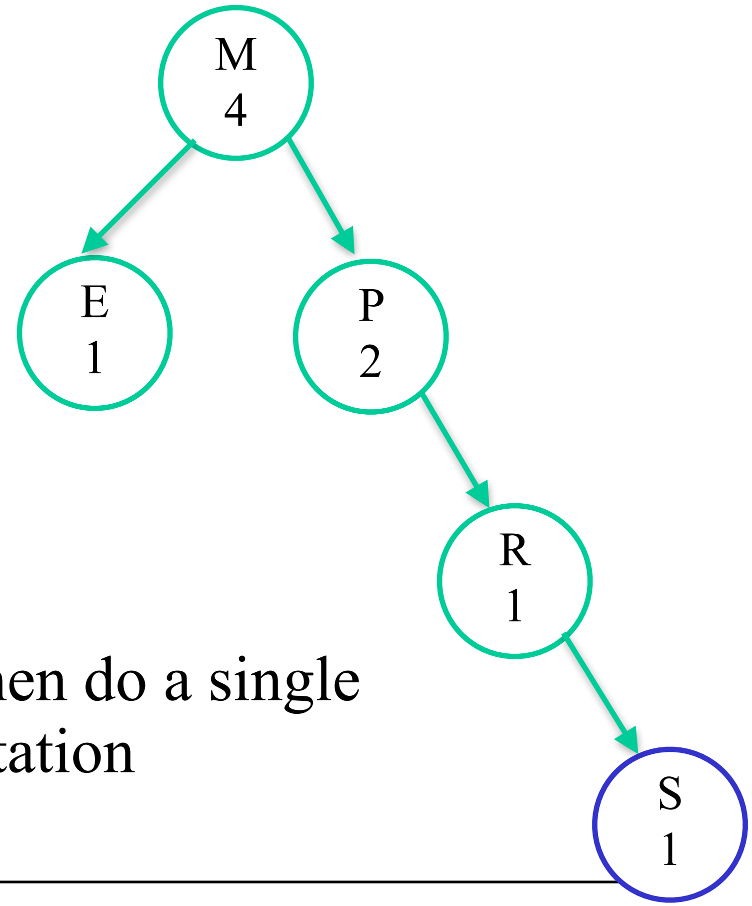
- Two types of rotation
- Single
  - left subtree of left node causes imbalance
  - right subtree of right node causes imbalance
- Double
  - right subtree of left node causes imbalance
  - left subtree of right node causes imbalance
    - The first rotation of a double puts the tree into position for a single rotation!

AVL Animation

Double Rotation

First rotate across the point imbalance

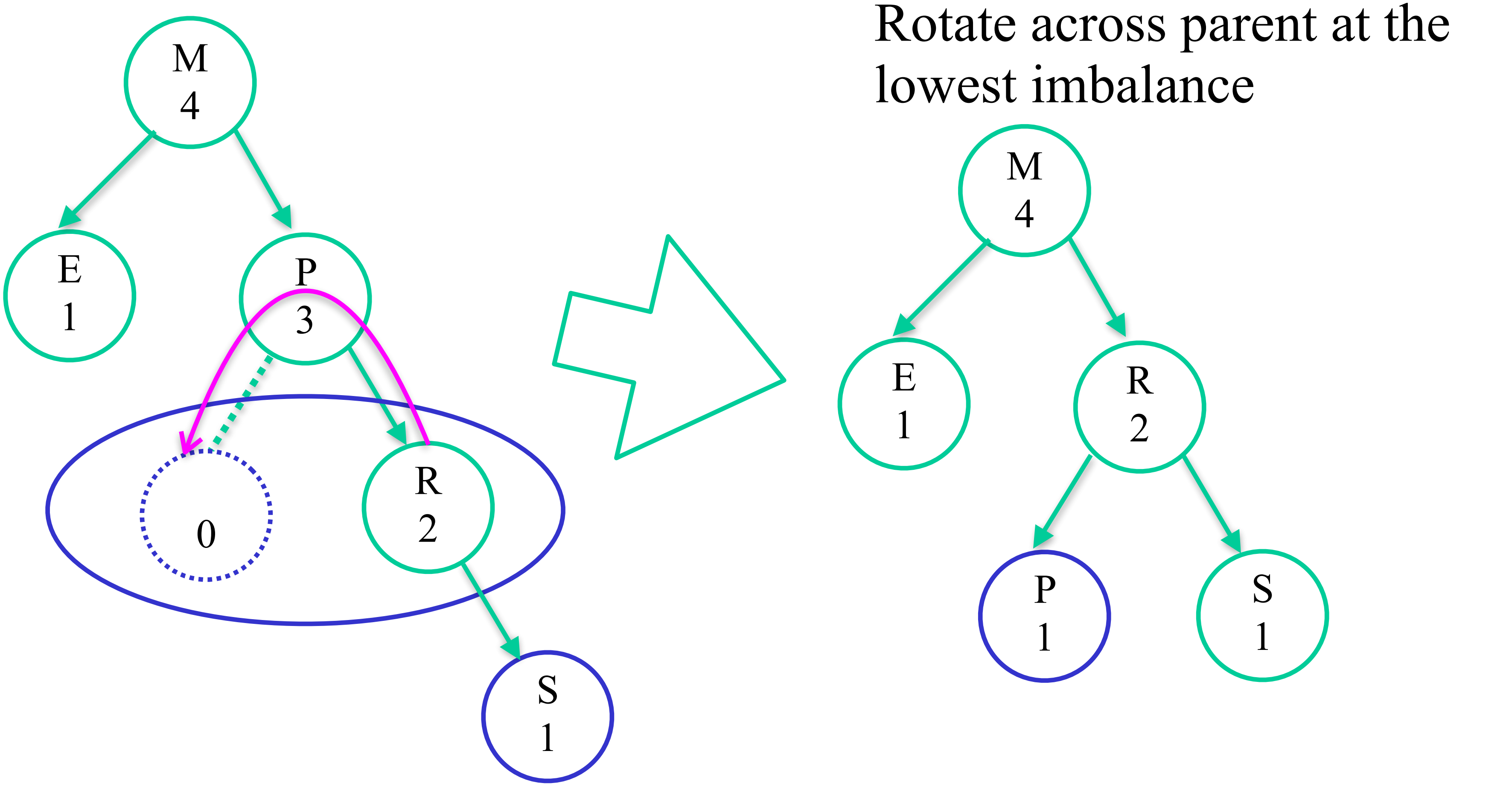




Then do a single rotation

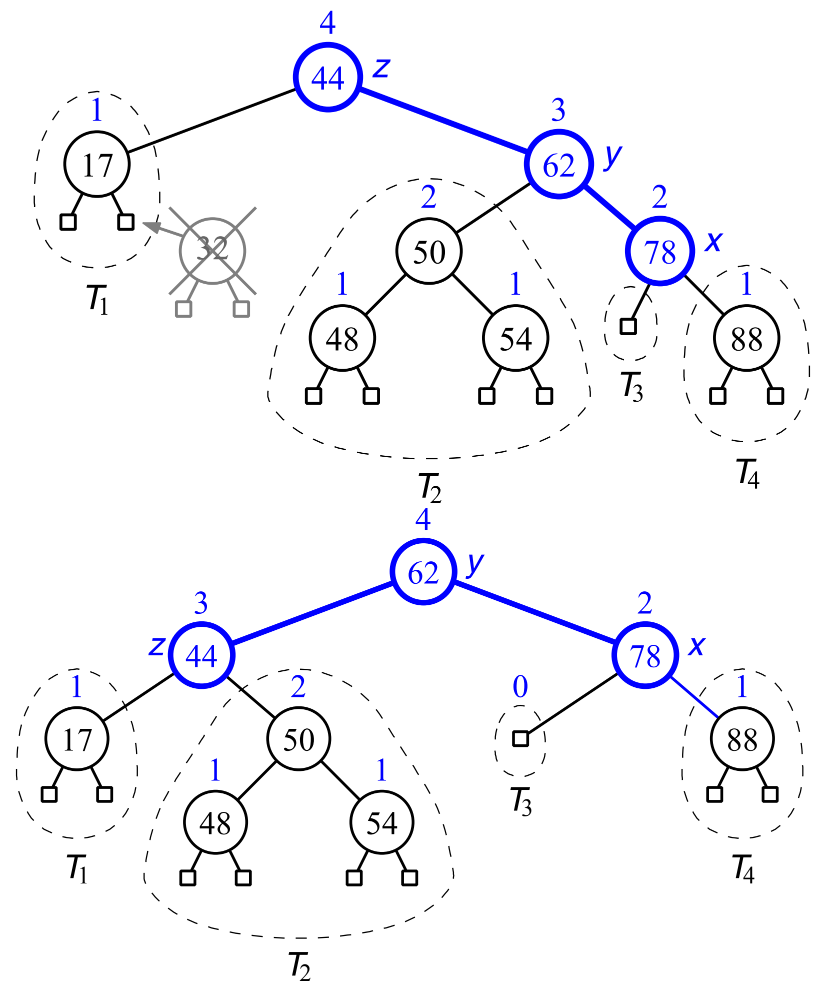
Single Rotation

Rotate across parent at the lowest imbalance



Deletion

- Deletion removes a node with 0 or 1 child
  - recall deletion from binary tree for node with 2 children.
- Deletion may reduce the height of parent
- Rotate to rebalance just like insertion
- Fix avlHeight
- May in case of ties, choose a single rotation.



$O(log n)$ Rotations

- Unlike insertion where rotation of the nearest unbalanced ancestor restores the balance globally
- On deletion, rotation of the nearest unbalanced ancestor only guarantees balance locally to the subtree
- Worst-case requires $O(log n)$ rotations up the tree to restore balance globally

Doing AVL

insert	100
insert	200
insert	300
insert	400
insert	500
insert	600
insert	700
insert	800
insert	900
insert	750
insert	1000
insert	850
delete	400
delete	300
delete	200
delete	700
delete	500

Mini-Lab
AVL tree practice

Show the BST tree and each AVL rotation (if needed) to keep a BST an AVL tree

insert	1000
insert	500
insert	750
insert	625
insert	560
insert	590
insert	400
insert	300
insert	600
insert	200
delete	560
delete	590