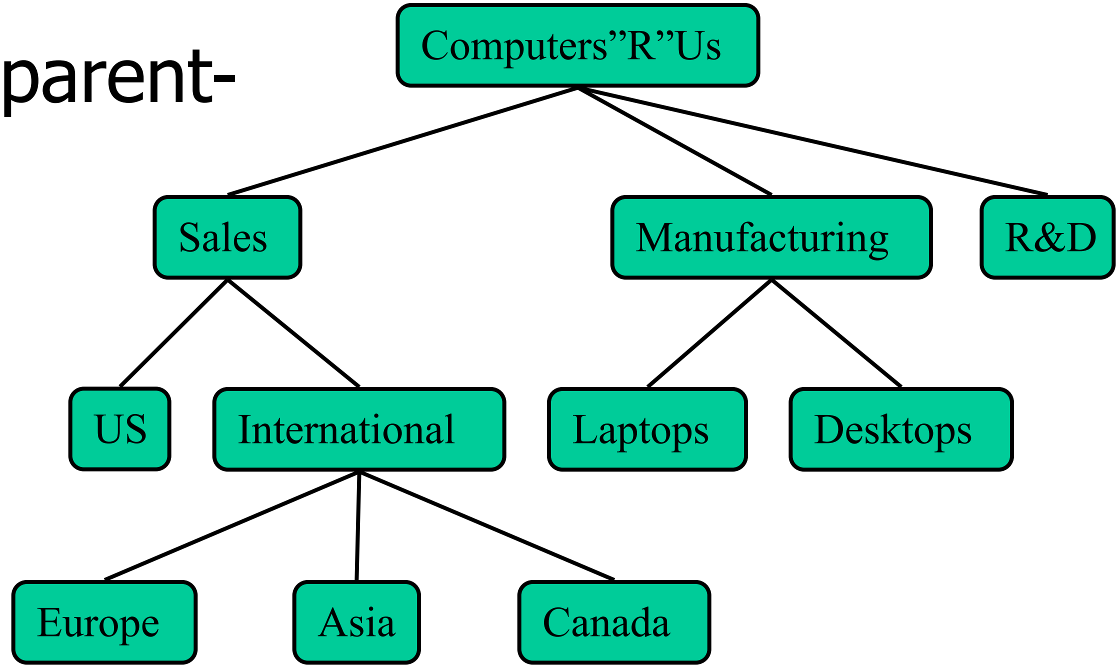
CS206

Trees

Tree

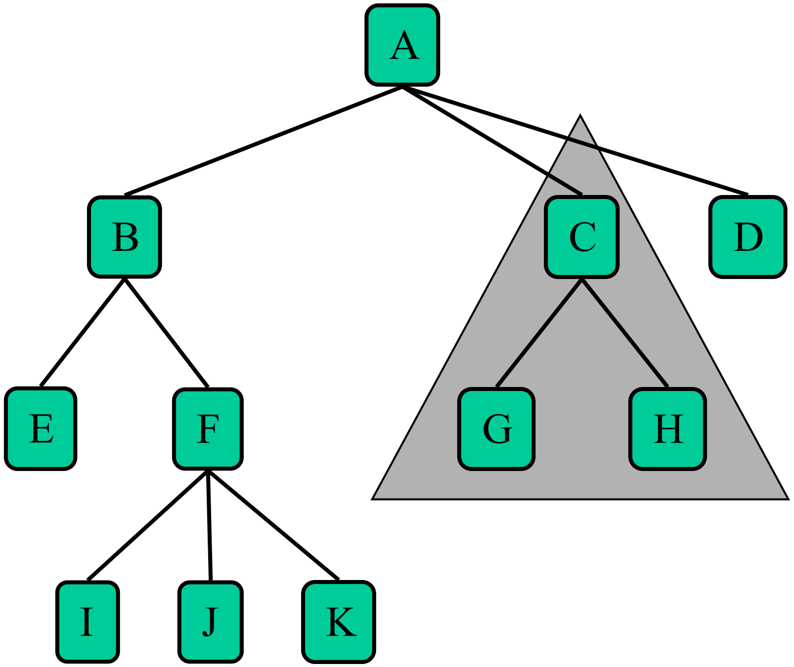
- A tree is an abstract model of a hierarchical structure
- Nodes have a parent-child relation
- NO LOOPS!



Terminology

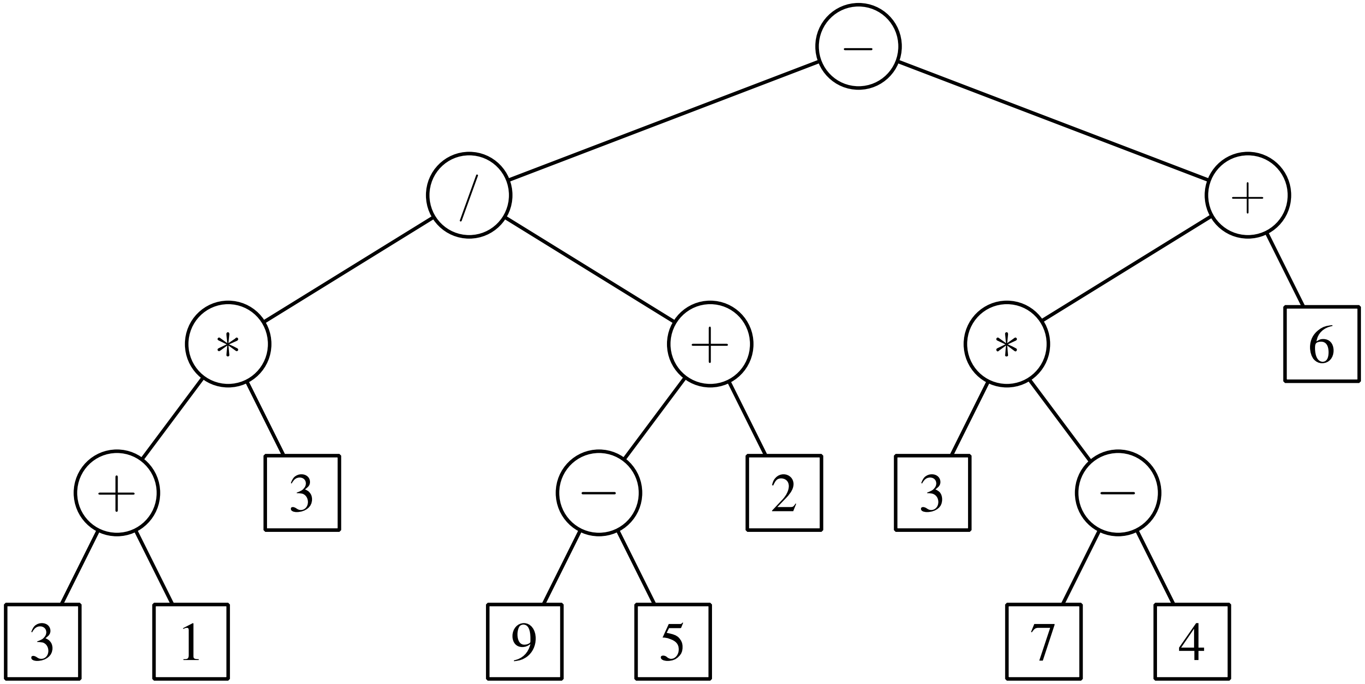
- root: no parent – A
- external node/leaf: no children – E, I, J, K, G, H, D
- internal node: - node with at least one child - A, B, C, F
- ancestor/descendent
- depth - # of ancestors
- Height - max depth

- Subtree: tree consisting of a node and its descendants



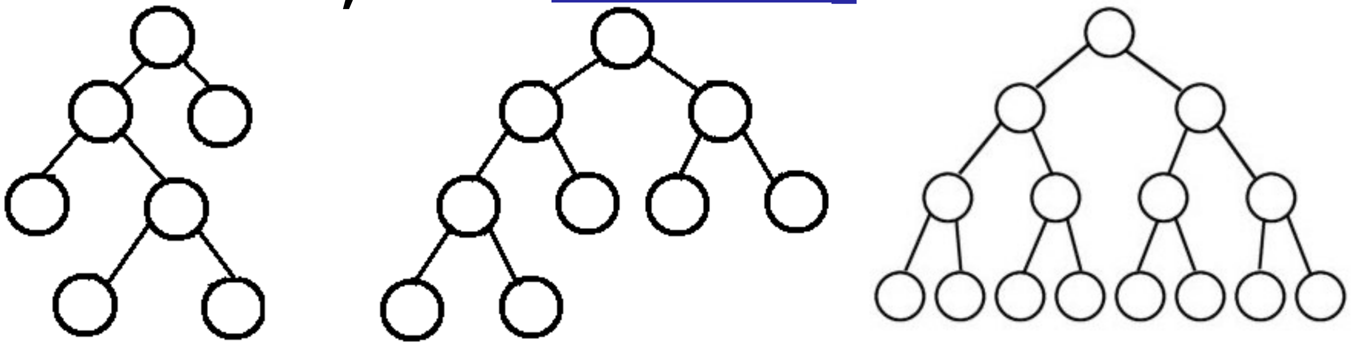
Binary Tree

- An ordered tree with every node having at most two children – left and right



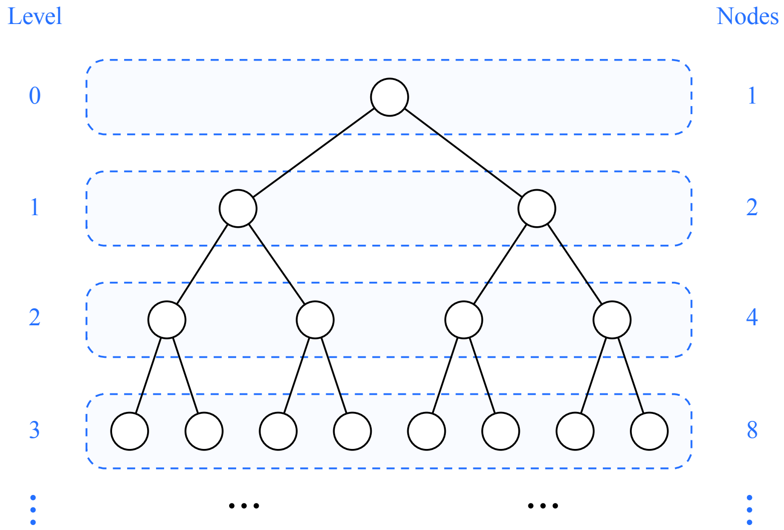
Type of Binary Trees

- A binary tree is **proper** (or full) if each node has zero or two children
- A binary tree is **complete** if every level (except possibly the last) is filled
- If a complete binary tree is filled at every level, it is **perfect**



Binary Tree Properties

- Let n denote the number of nodes and h the height of a binary tree
  - $h + 1 \le n \le 2^{h+1} - 1$
  - $\log(n + 1) - 1 \le h \le n - 1$
- Height of a binary tree is usually $O(logn)$ of the max number of nodes — true worst case O(n)

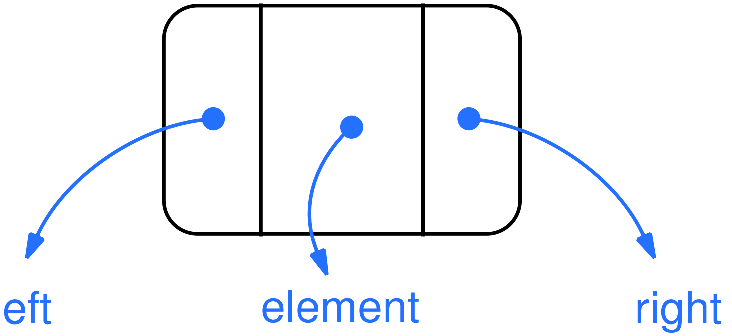


Interface

```java
public interface TreeInterface<B>
{
    int size();
    int height();
    boolean isEmpty();
    boolean contains(B element);
    void insert(B element);
    B remove(B element);
}
```

Implementation

```
private class Node {
    E payload;
    Node right;
    Node left;

    public Node(E e) {
        payload=e;
        right=null;
        left=null;
    }
    public String toString() {
        return payload.toString();
    }
}
```



This looks a lot like a doubly linked list!!
So, is a doubly linked list a tree?

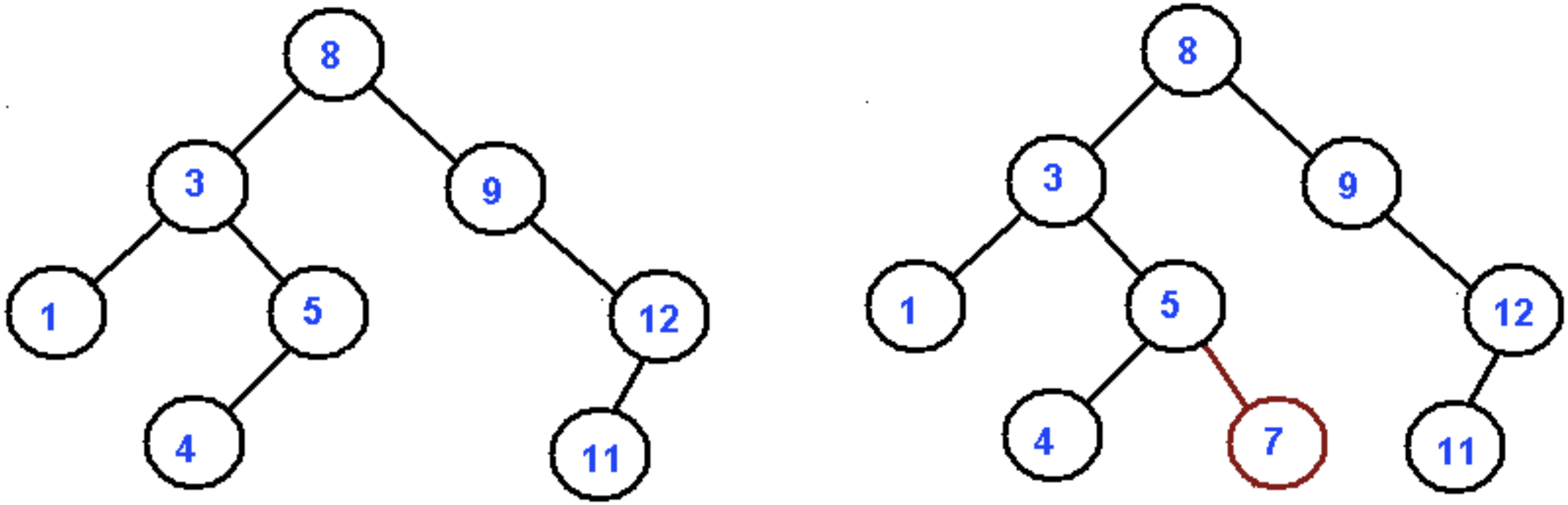
Class

```
public class LinkedBinaryTree<E
extends Comparable<E>> implements
TreeInterface<E>
{
    /** The number of elements in the
tree */
    private int size;

    /** The root of the tree */
    private Node root;
```

Class name violates Encapsulation!

Insertion

- smaller to the left, bigger to the right



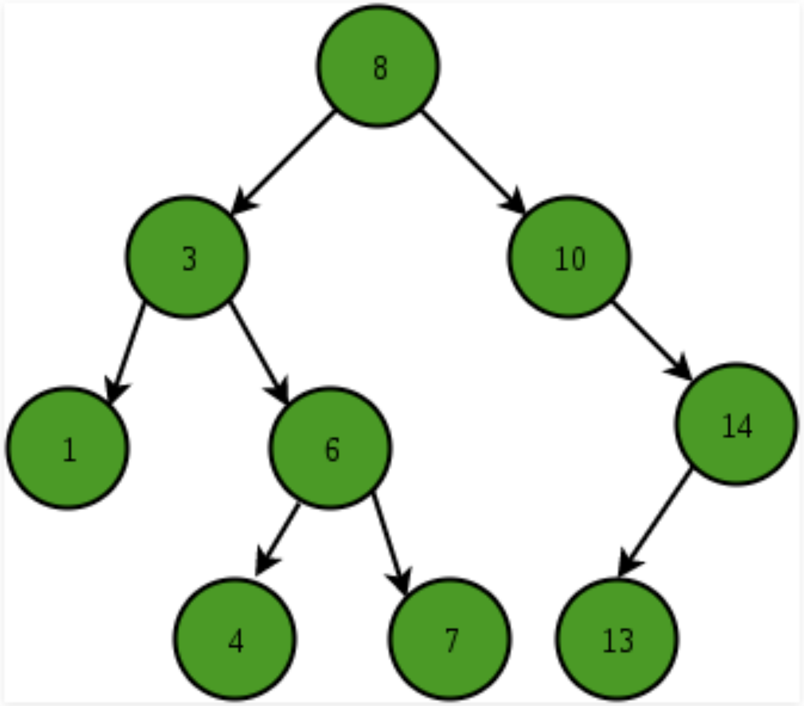
Following this pattern creates a “Binary Search Tree”

Draw some Binary Trees

- 11, 6, 8, 19, 4, 10, 5, 17, 43, 49, 31
- 6, 19, 10, 5, 43, 31, 11, 8, 4, 17, 49
- 4, 5, 6, 49, 43, 31, 19, 10, 11, 8, 17
- 17, 31, 8, 19, 43, 11, 5, 49, 10, 6, 4

contains

- `boolean contains(E element);`
- returns true if found in the tree, false otherwise



Contains Algorithm

- compare with root of **current subtree**
    - root is empty – return false
    - root == element – return true
    - root < element – recurse on right child
    - root > element - recurse on left child

Pseudo Code

```
findRec(root, key):
  if root == null:
    return false
  if root.key == key:
    return true
  if root.key > key:
    return findRec(root.left, key)
  else
    return findRec(root.right, key)
```

Recursive Helper Method

- The signature of `contains` doesn’t allow any `Node` references (it cannot since `Node` is private)
- so define a private , recursive “helper” method.

```
public boolean contains(E element) {
   if (root==null) return false;
   return containsUtil(root, element)!=null;
}
private Node containsUtil(Node treepart, E toBeFound)
{
      … }
```

live write

Unordered Contains

- Suppose that you did not know relation among children
  - So thing being looked for could be either left or right
  - How would you change containsUtil function
    - Would a tree be a useful structure in this case?

insert

- `void insert(E element);`
- new node is always inserted as a leaf
- inserts to
  - left subtree if element is smaller than subtree root
  - right subtree if larger
- Pre-case: if root=null then root=new Node

```
public void insert(E element) {
    if (root==null) {
      root=new Node<E>(element);
      size = 1;
    } else
      insertUtil(root, element);
  }
```

Pseudo Code for recursion

```
insertUtil(node, element):
    if element==node.payload
        return;
    if node.payload > element:
      if node.left==null
        node.left=new Node(payload)
      else
        insertUtil(node.left,element);
    else
        // same but for right
```

InsertUtil

```
private void insertUtil(Node treepart, E toBeAdded) {
 … }
```

mini-lab write

Height / maxDepth

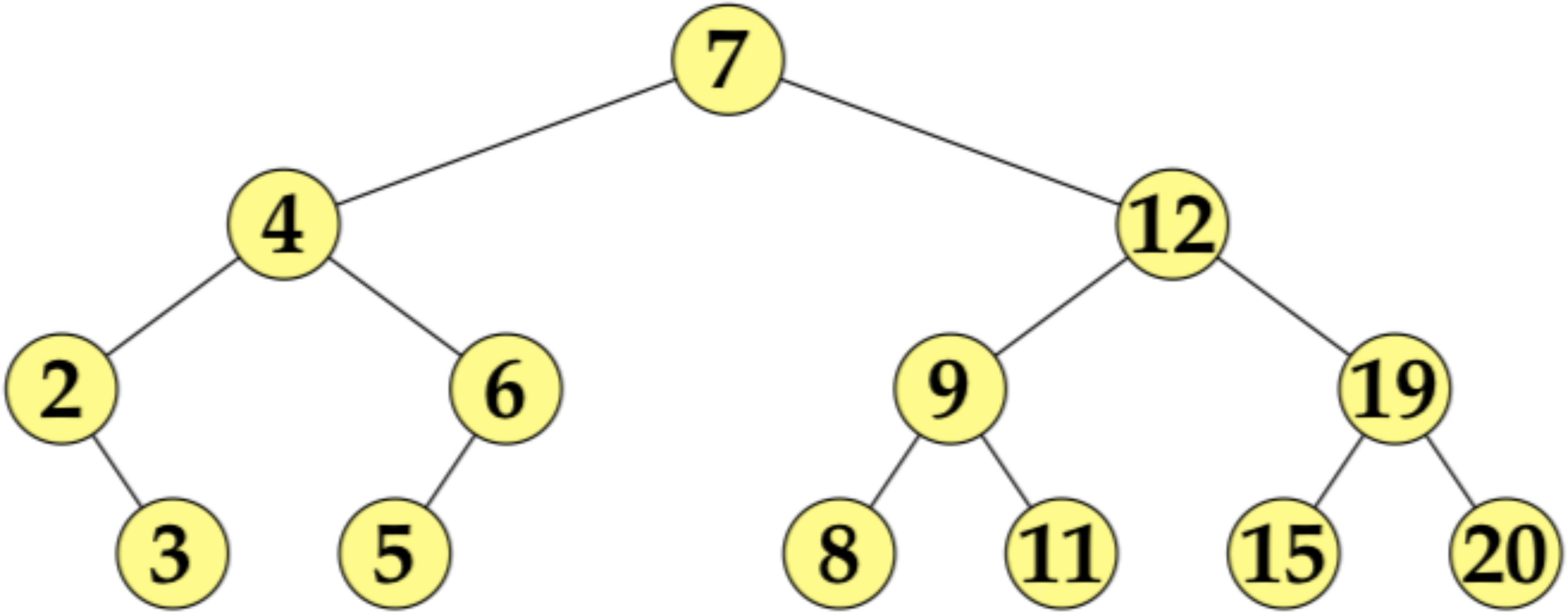
Again, using a recursive helper method

```
@Override
public int maxDepth()
{
   return maxDepthUtil(root, 1);
}

int maxDepthUtil(Node n, int depth) {
   …}
```

size() without size

```
public int sizeAlt() {
    return iSize(root);
}
private int sizeAltUtil(Node treepart) {
    if (treepart==null) return 0;
    return 1 + sizeAltUtil(treepart.left) +
               sizeAltUtil(treepart.right);
}
```

Traversals / Printing



Postorder traversal

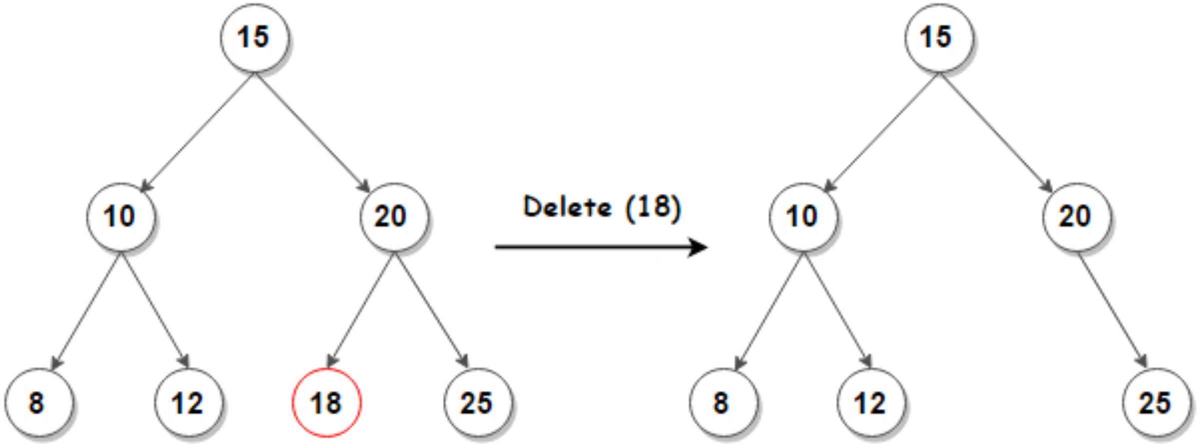
```
public void printPostOrder() {
    iPrintPostOrder(root, 0);
    System.out.println();
}

private void iPrintPostOrder(Node treePart, int depth) {
    if (treePart==null) return;
    iPrintPostOrder(treePart.left, depth+1);
    iPrintPostOrder(treePart.right, depth+1);
    System.out.print("["+treePart.payload+","+depth+"]");
}
```

Remove

- `boolean remove(E element);`
- returns true if element existed and was removed and false otherwise
- Cases
  - element not in tree
  - element is a leaf
  - element has one child
  - element has two children

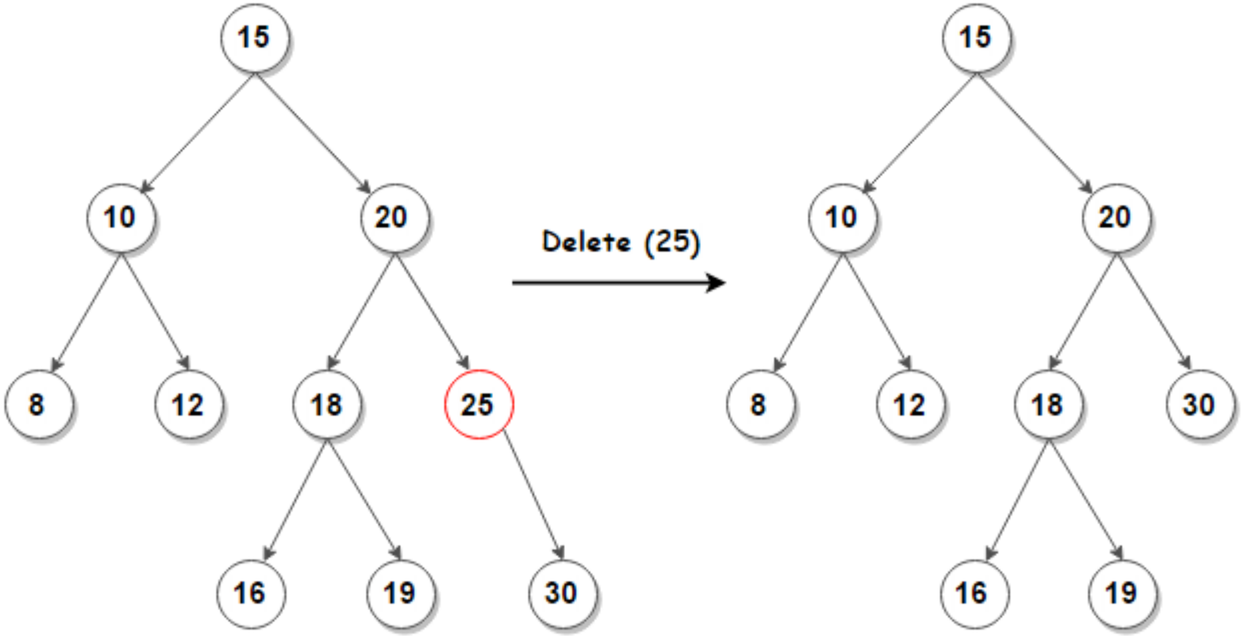
Leaf

- Just delete



One child

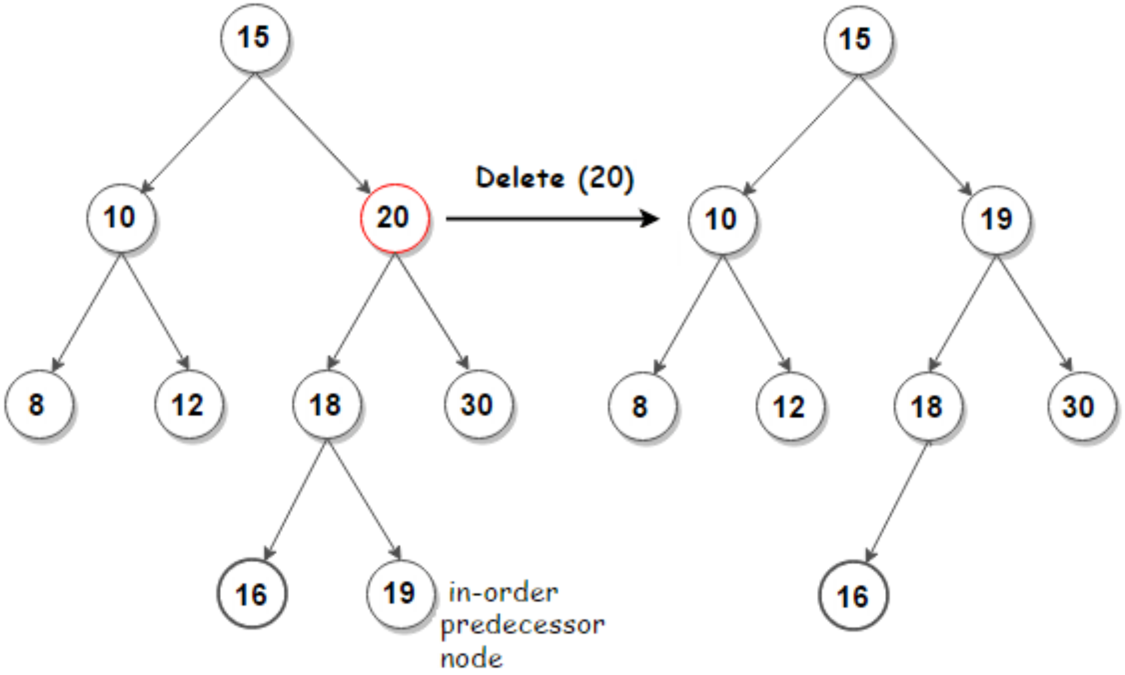
- Replace with child – skip over like in linked list



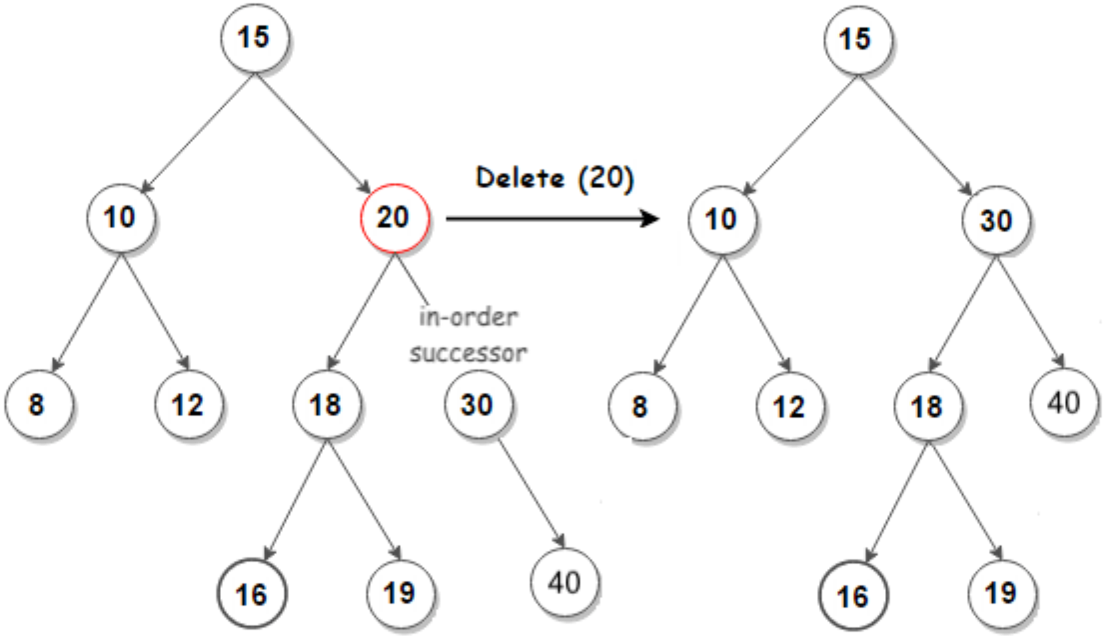
Two Children

- Replace with in-order predecessor or in-order successor
- in-order predecessor
  - rightmost child in left subtree
  - max-value child in left subtree
- in-order successor
  - leftmost child in right subtree
  - min-value child in right subtree

Replace with Predecessor



Replace with Successor



mini-lab exercise

- Complete the implementation of insertUtil using pencil and paper is OK.
- Strive to be correct
- Think.
  - Draw pictures of trees and what you want your code to do.
- Take a picture of your code and send it to gtowell206@cs.brynmawr.edu

```
private Node containsUtil(Node treepart, E toBeFound)
 {
 if (treepart==null) return null;
 int cmp = treepart.element.compareTo(toBeFound);
 if (cmp==0)
 {
     return treepart;
 }
 else if (cmp<0)
 {
     return containUtil(treepart.left, toBeFound);
 }
 else // cmp>0
 {
     return containUtil(treepart.right, toBeFound);
 }
 }
```