Priority Queues

cs206

lec 19

April 7

Priority Queue

- A queue that maintains order of elements according to some priority
  - Removal order, not general order
    - the rest may or may not be sorted

Key Value Pairs

- Priority Queues are usually described as being on Key-Value Pairs
    - akin to Hashtables
- Priority queues are ordered by the key, which may be:
  - derived from the Value element (which may be a large, complex object)
    - one field
    - combination of fields
  - independent of Value element
    - for example: insertion time
- best practice is make keys implement `Comparable` relation between keys using `compareTo`
- Keys ideally:
  - are unique
    - how to handle duplicate keys?
  - have a natural ordering.
    - Contrast to hashtables in which key ordering is irrelevant

Priority Queues in real world

- Homework
  - key= f(due date, difficulty, annoyance)
- Others items in priority queues
  - what is the key?

PriorityQueue Interface

```
public interface QueueInterface<Q> {
    boolean isEmpty();
    int size();
    boolean offer(Q q);
    Q poll();
    Q peek();
}
```

```
public interface PriorityQInterface<K extends Comparable<K>, V> {
    boolean isEmpty();
    int size();
    boolean offer(K key, V value);
    V poll();
    V peek();
}
```

AbstractPriorityQueue

```
public abstract class AbstractPriorityQueue <K extends Comparable<K>, V> implements PriorityQInterface<K,V> {
    enum Ordering { ASCENDING, DESCENDING }
    protected Ordering order;
    protected class Pair<L extends Comparable<L>, W> implements Comparable<Pair<L,W>> {
        /** Hold the key */
        final L theK;
        /** Hold the  value*/
        final W theV;
        public Pair(L kk, W vv) {
            theK = kk;
            theV = vv;
        }
        @Override
        public int compareTo(Pair<L, W> o) {
            if (Ordering.ASCENDING == order || Ordering.MIN==order)
                return theK.compareTo(o.theK);
            return o.theK.compareTo(theK);
        }

        public String toString() {
            return "{{"+theK+" " +theV+"}}";
        }

    }
```

PQ Implementation

- Questions:
  - How to store keys and values
    - handling of duplicate keys
  - Is the storage:
    - ordered?
    - size bound?

(Internally Unordered) Priority Q

```
public class PriorityQueue<K extends Comparable<K>, V>  extends AbstractPriorityQueue<K,V> {
    private static int CAPACITY = 200;
    private Pair<K,V>[] pqStore;
    private int size;
    public PriorityQueue() {
        this(Ordering.MIN);
    }
    public PriorityQueue(Ordering order) {
        this.order=order;
        pqStore = (Pair<K,V>[]) new Pair[CAPACITY];
        this.size=0;
    }
    public int size() {
        return size;
    }
    public boolean isEmpty() {
        return size==0;
    }
    public boolean offer(K newK, V newV) {
        if (size==CAPACITY)
            return false;
        Pair<K,V> entry = new Pair<>(newK, newV);
        pqStore[size]=entry;
        size++;
        return true;
    }
```

peek & poll

```
public V peek() {
        int lmin = getNext();
        if (lmin<0)
            return null;
        Pair<K,V> entry = pqStore[lmin];
        if (entry==null) return null;
        return entry.theV;
    }
```

```
   public V poll() {
        if (size==0) return null;
        if (size==1) {
            Pair<K,V> entry = pqStore[0];
            pqStore[0]=null;
            size=0;
            return entry.theV;
        }
        int lmin = getNext();
        Pair<K,V> entry = pqStore[lmin];
        remove(lmin);
        size--;
        return entry.theV;
    }
```

getNext(), remove(lmin)

`write them.`

Example

```
PriorityQueue<Integer, String> pq = new PriorityQueue<>(Ordering.MIN);
        pq.offer(1,"Jane");
        pq.offer(10,"WET");
        pq.offer(5, "WAS");
        System.out.println(pq.poll());
        System.out.println(pq.poll());
        System.out.println(pq.poll());
        System.out.println();

        pq = new PriorityQueue<>(Ordering.MAX);
        pq.offer(1,"Jane");
        pq.offer(10,"WET");
        pq.offer(5, "WAS");
        System.out.println(pq.poll());
        System.out.println(pq.poll());
        System.out.println(pq.poll());
```

(Internally Ordered) Priority Q

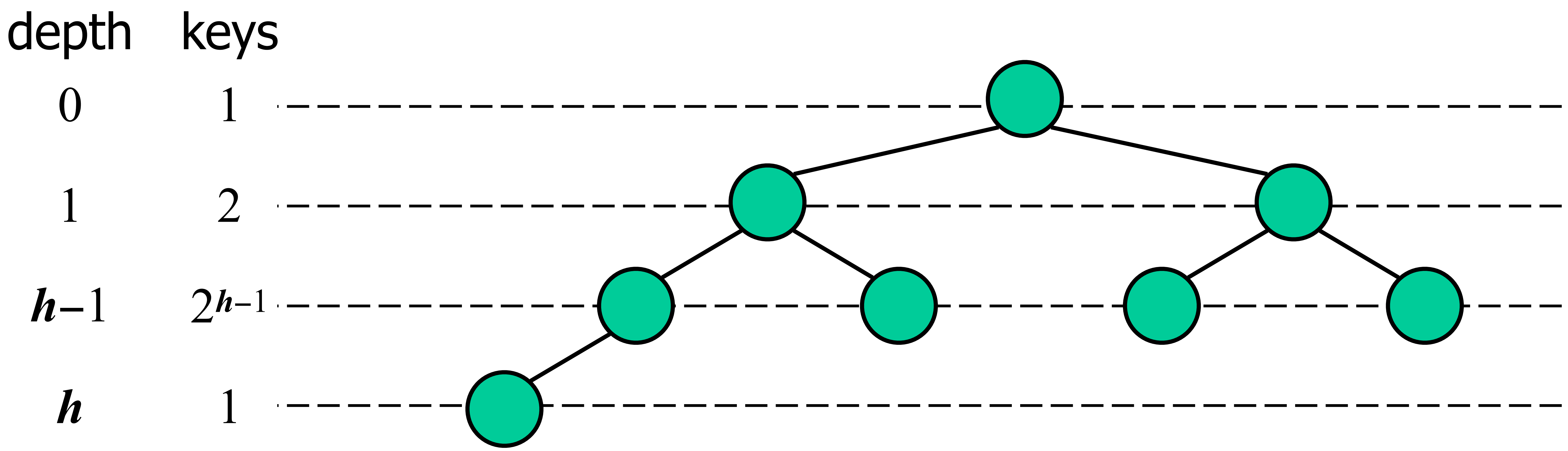
```
public class PriorityQueueSAL<K extends Comparable<K>, V> extends AbstractPriorityQueue<K,V> {
   final private SAL<Pair<K,V>> pqStore;
   public PriorityQueueSAL() { this(Ordering.ASCENDING); }
    public PriorityQueueSAL(Ordering order) {
        this.order=order;
        pqStore = new SAL<>(SAL.Ordering.DESCENDING);
    }
   public int size() {
        return pqStore.size();
    }
   public boolean isEmpty() {
        return pqStore.isEmpty();
   }
    public boolean offer(K newK, V newV) {
        pqStore.add(new Pair<>(newK, newV));
              return true;  // Note that this always succeeds, so always return true.
    }
   public V poll() {
        if (isEmpty())
            return null;
        Pair<K,V> p = pqStore.getAndRemove(pqStore.size()-1);
        return p.theV;
    }
```

Binary Heap

- A heap is a "binary tree" storing keys at its nodes and satisfying:
  - heap-order: for every internal node v other than root, $key(v) \geq key(parent(v))$
  - "complete binary tree": let h be the height of the heap
    - Heap is filled from top down and within a level from left to right.
      - at depth h, the leaf nodes are in the leftmost positions
      - last node of a heap is the rightmost node of max depth

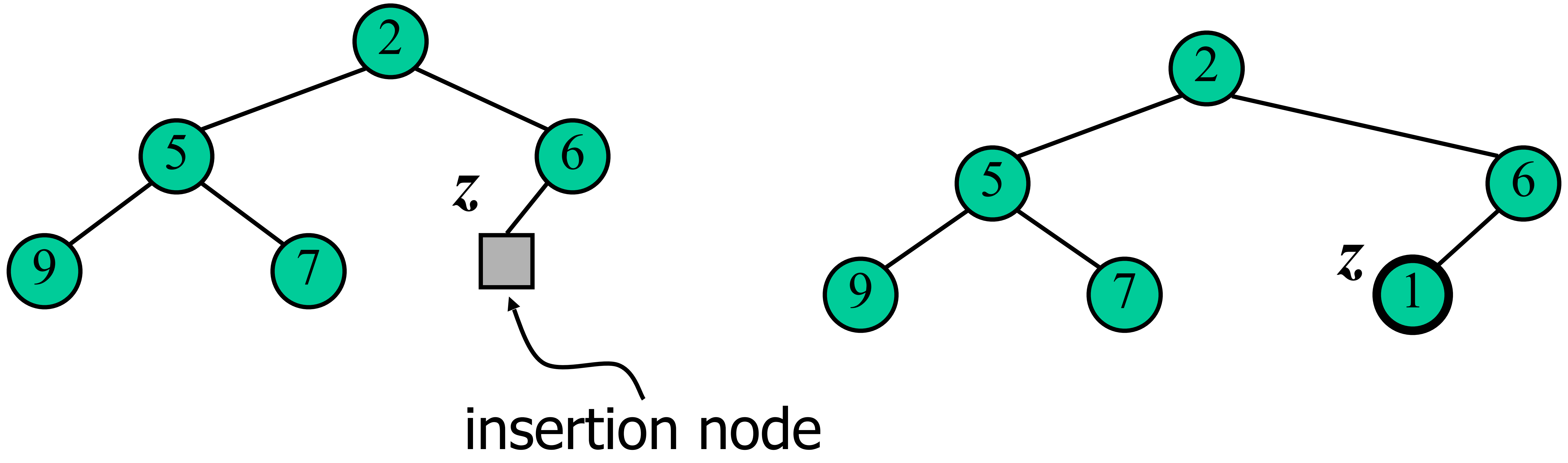
Height of a Heap

- A binary heap storing n keys has a height of $O(\log_2 n)$



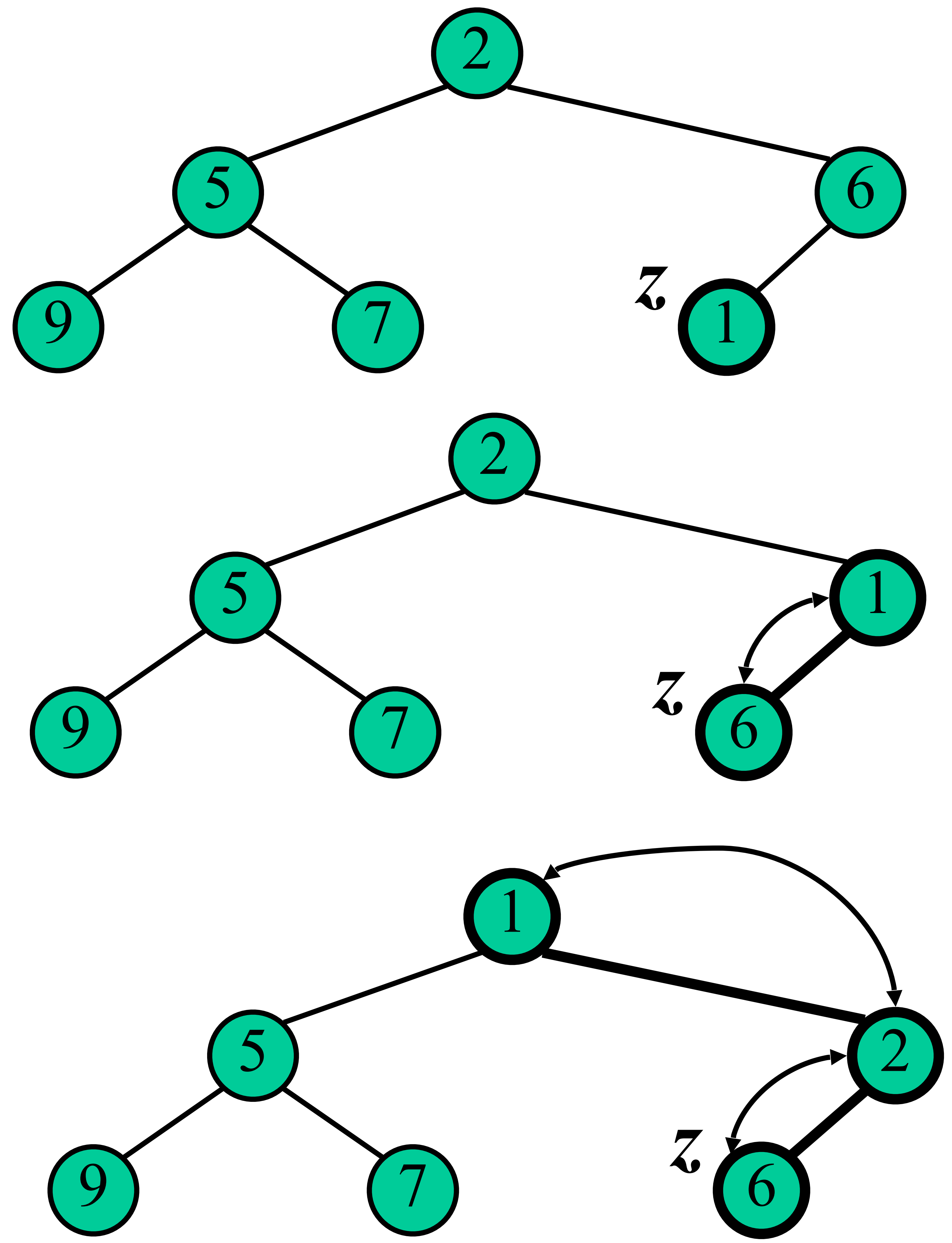
Insertion into a Heap

- Insert as new last node
- Need to restore heap order



Upheap

- Restore heap order
  - swap upwards
  - stop when finding a smaller parent
  - or reach root
- $O(\log n)$



Priority Queue using Heaps

startup

```
public class PriorityQHeap<K extends Comparable<K>, V> implements PriorityQInterface<K,V>
{

/** The default size of the heap.  This corresponds to a max depth or 10. */
    private static final int CAPACITY = 1032;
    /** The array that holds the heap. */
    private Pair<K,V>[] backArray;
    /** The number of items actually in he heap. */
    private int size;
    /** The way in which the heap is ordered */
    final private Ordering order;

   public PriorityQHeap() {
        this(Ordering.MIN, CAPACITY);
    }

   public PriorityQHeap(Ordering order, int capacity) {
        this.order=order;
        backArray = new Pair[capacity];
    }
```

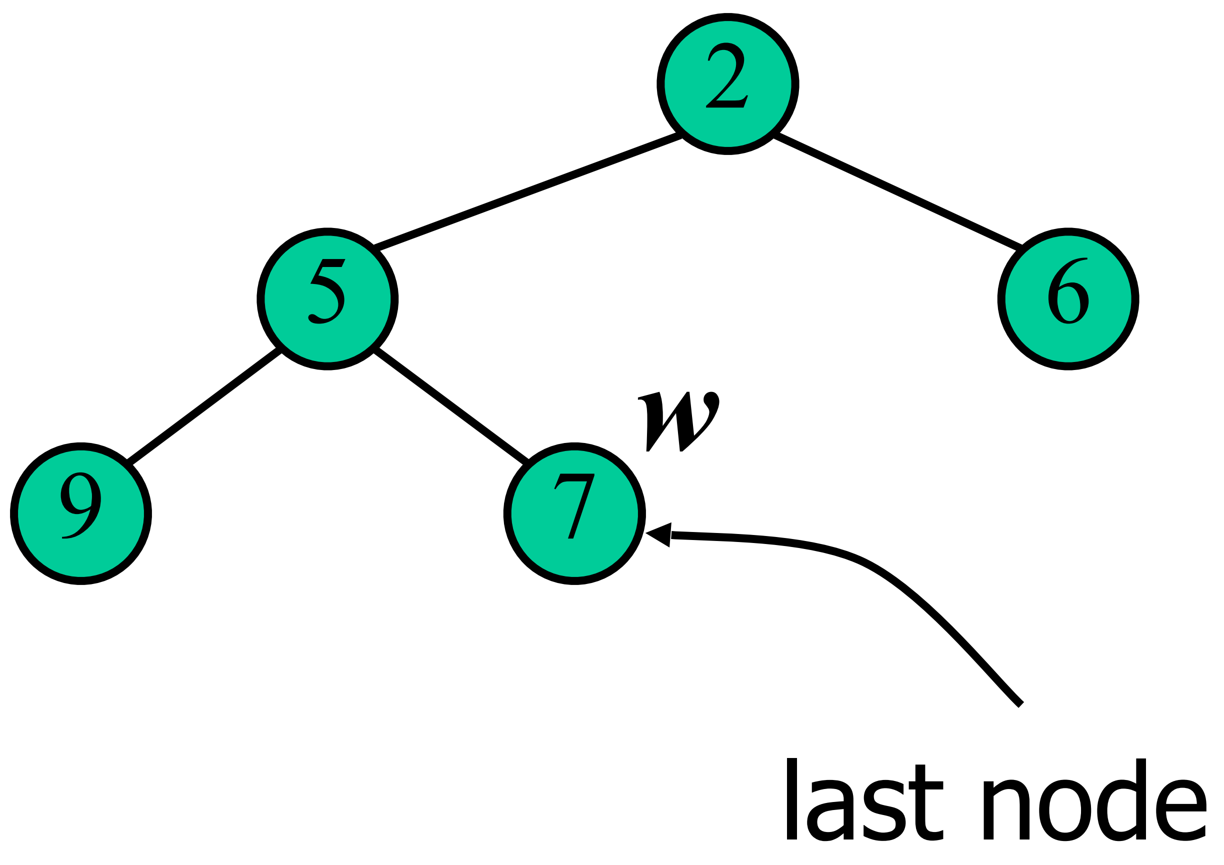
Heap Insertion

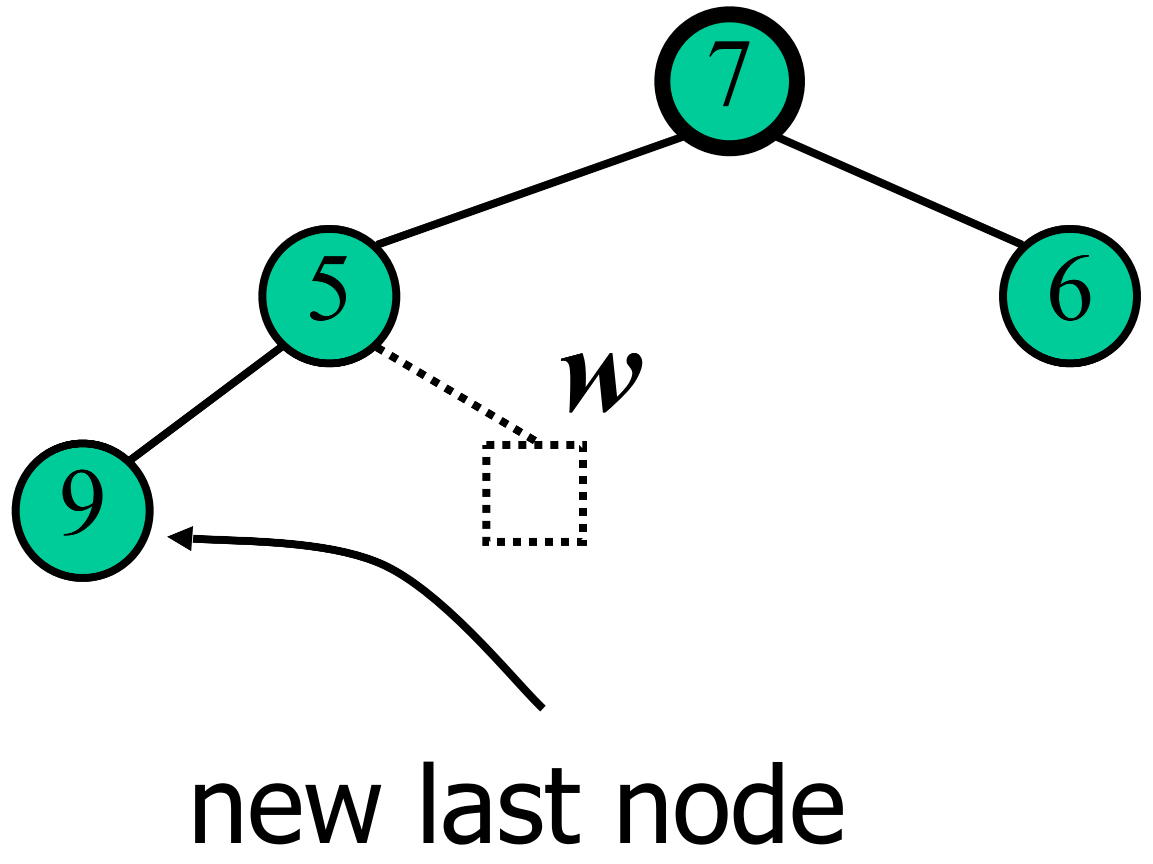
Priority Queue offer method

```
public boolean offer(K key, V value) {
    if (size>=(backArray.length-1))
        return false; // no space in the array
    // put in at end
    int loc = size++;
    backArray[loc] = new Pair<K,V>(key, value);
    // up heap
    int upp = (loc-1)/2;
    while (loc!=0) {
        if (0 > backArray[loc].doCompare(backArray[upp])) {
            // swap and climb
            Pair<K,V> tmp = backArray[upp];
            backArray[upp] = backArray[loc];
            backArray[loc] = tmp;
            loc = upp;
            upp = (loc-1)/2;
            }
        else {
            break;
        }
    }
    return true;
    }
```

Poll

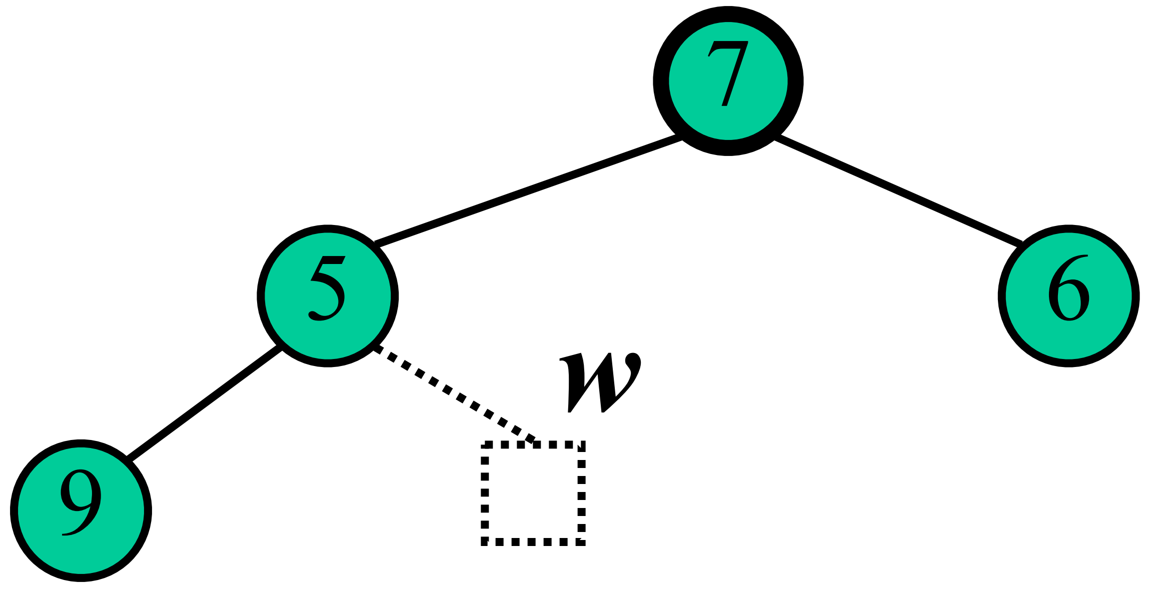
- Removing the root of the heap
  - Replace root with last node
  - Remove last node w
  - Restore heap order

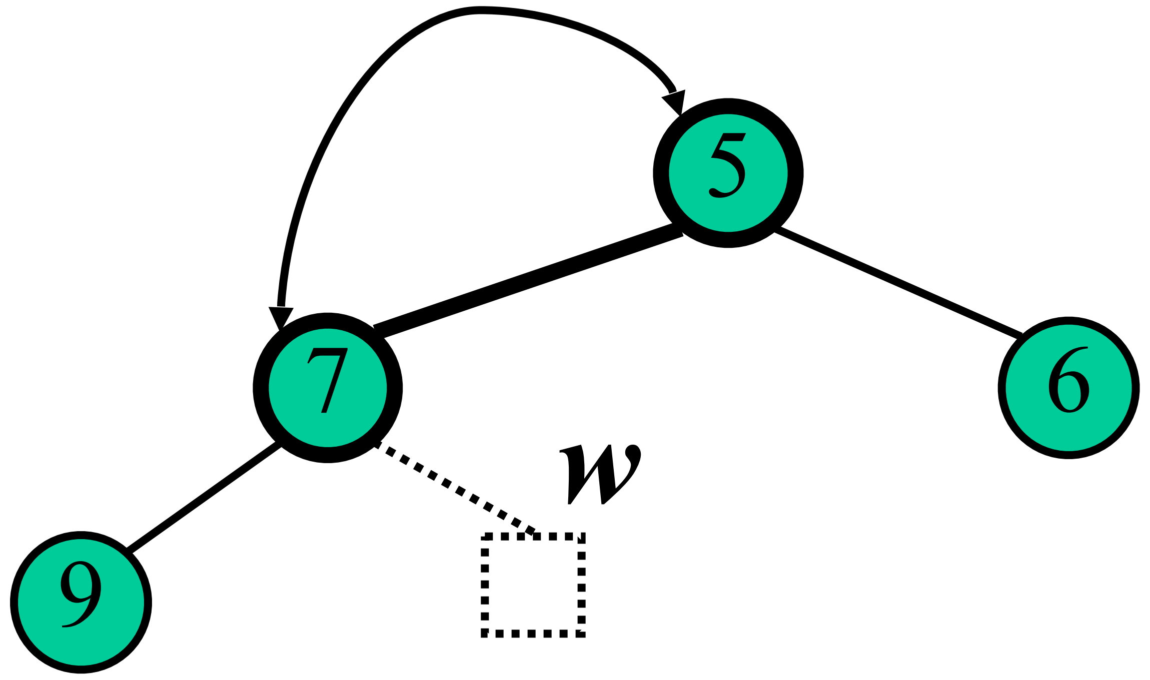




Downheap

- Restore heap order
  - swap downwards
  - swap with smaller child
  - stop when finding larger children
  - or reach a leaf
- $O(logn)$





Peek and Poll

```
@Override
    public V poll() {
        if (isEmpty())
            return null;
        Entry<K,V> tmp = backArray[0];
        removeTop();
        return tmp.theV;
    }

    @Override
    public V peek() {
        if (isEmpty())
            return null;
        return backArray[0].theV;
    }
```

Remove head item from Heap

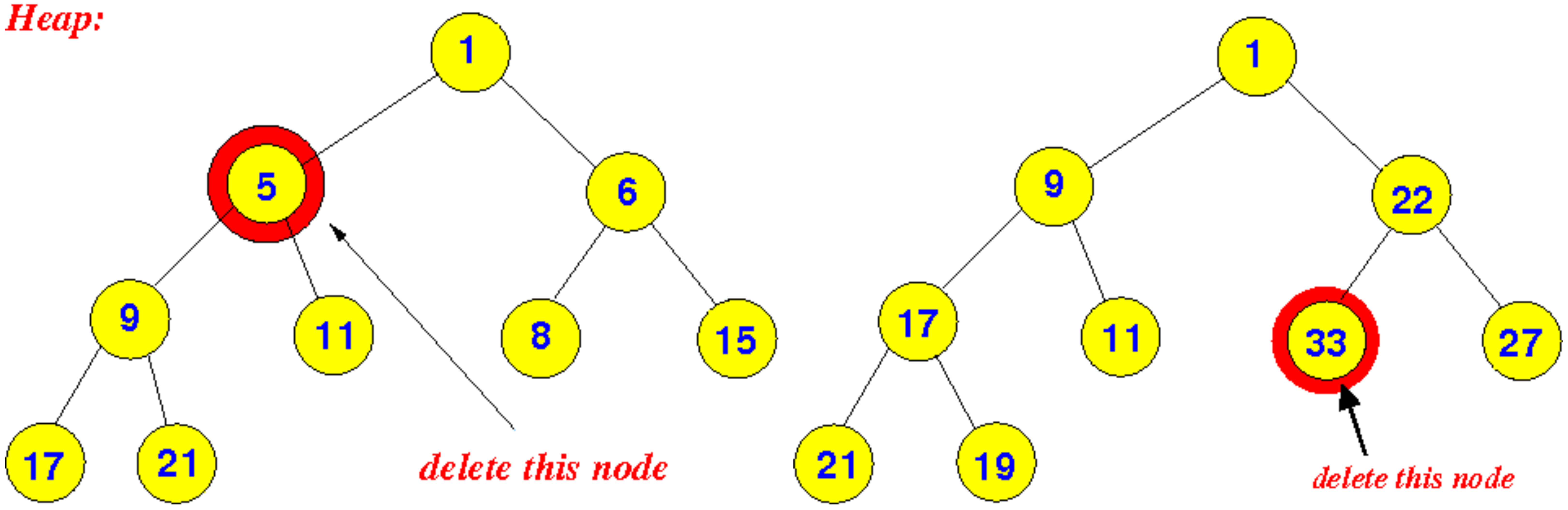
```
private void removeTop()
{
    backArray[0] = backArray[size-1];
    backArray[size-1]=null;
    size--;
    int upp=0;
    while (true)
    {
        int dwn;
        int dwn1 = upp*2+1;
        if (dwn1>size) break;
        int dwn2 = upp*2+2;
        if (dwn2>size) {  dwn=dwn1;
        } else {
            int cmp = backArray[dwn1].compareTo(backArray[dwn2]);
            if (cmp<=0)  dwn=dwn1;
            else dwn=dwn2;
        }
        if (0 > backArray[dwn].compareTo(backArray[upp]))
        {
            Pair<K,V> tmp = backArray[dwn];
            backArray[dwn] = backArray[upp];
            backArray[upp] = tmp;
            upp=dwn;            }
        else {  break;            } } }
```

Complexity Analysis

	Unordered	Ordered (using SAL)	Heap Based
offer			
peek			
poll			

General Removal

- swap with last node
- delete last node
- may need to upheap or downheap



Complexity Analysis

	Unordered	Ordered (using SAL)	Heap Based
offer			
peek			
poll			
General Removal			