TREES

Chapter 6

Trees - Introduction

- All previous data organizations we've studied are linear—each element can have only one predecessor and successor
- Accessing all elements in a linear sequence is O(n)
- Trees are nonlinear and hierarchical
- Tree nodes can have multiple successors (but only one predecessor)

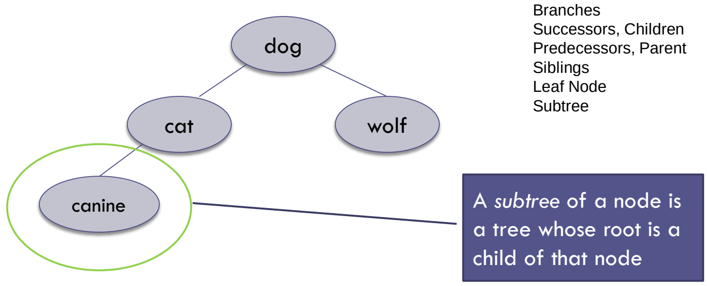
Tree Terminology and Applications

Section 6.1

Tree Terminology (cont.)

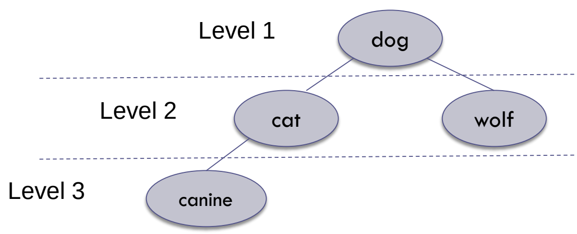
A tree consists of a collection of elements or nodes, with each node linked to its successors

Node, Link
Root
Branches
Successors, Children
Predecessors, Parent
Siblings
Leaf Node
Subtree



Tree Terminology (cont.)

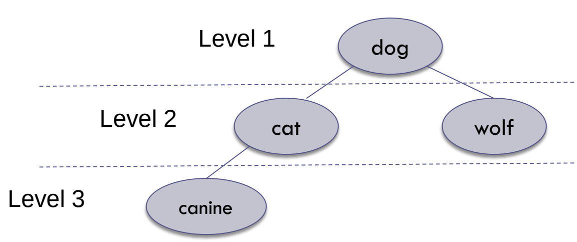
A tree consists of a collection of elements or nodes, with each node linked to its successors



The *level of a node* is its distance from the root plus 1

Tree Terminology (cont.)

A tree consists of a collection of elements or nodes, with each node linked to its successors



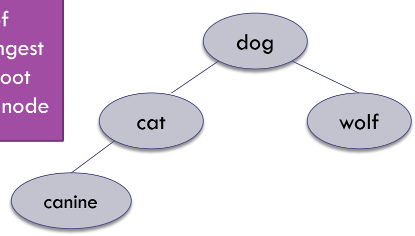
The *level of a node* is defined recursively

- If node *n* is the root of tree T, its level is 1
- If node *n* is not the root of tree T, its level is 1 + the level of its parent

Tree Terminology (cont.)

A tree consists of a collection of elements or nodes, with each node linked to its successors

The *height of a tree* is the number of nodes in the longest path from the root node to a leaf node

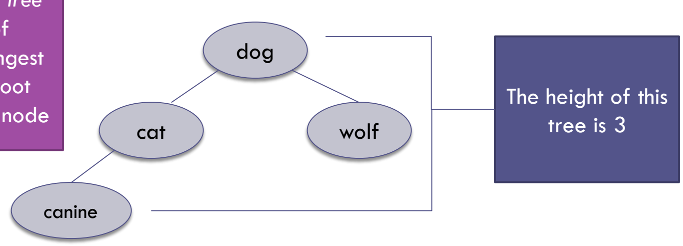


Node, Link
Root
Branches
Successors, Children
Predecessors, Parent
Siblings
Leaf Node
Subtree
Level
Height

Tree Terminology (cont.)

A tree consists of a collection of elements or nodes, with each node linked to its successors

The *height of a tree* is the number of nodes in the longest path from the root node to a leaf node



Common Types of Trees

- Binary Tree
- Expression Trees
- Huffman Trees
- Binary Search Trees
- Many many more!

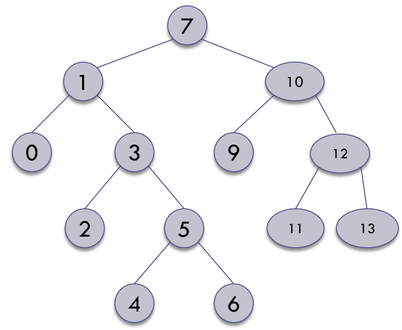
Binary Trees

- In a binary tree, each node has two subtrees
- A set of nodes T is a binary tree if either of the following is true
  - T is empty
  - Its root node has two subtrees, T_L and T_R, such that T_L and T_R are binary trees

  (T_L = left subtree; T_R = right subtree)

Full, Perfect, and Complete Binary Trees

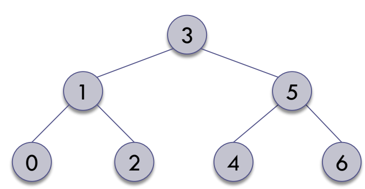
- A full binary tree is a binary tree where all nodes have either 2 children or 0 children (the leaf nodes)



Node, Link
Root
Branches
Successors, Children
Predecessors, Parent
Siblings
Leaf Node
Subtree
Binary Tree
Binary Search Tree
Full Binary Tree

Full, Perfect, and Complete Binary Trees (cont.)

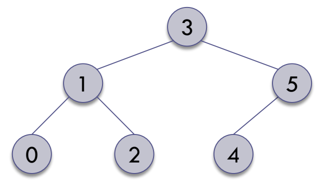
- A *perfect binary tree* is a full binary tree of height *n* with exactly $2^n - 1$ nodes
- In this case, $n = 3$ and $2^n - 1 = 7$



Node, Link
Root
Branches
Successors, Children
Predecessors, Parent
Siblings
Leaf Node
Subtree
Binary Tree
Binary Search Tree
Full Binary Tree
Perfect Binary Tree

Full, Perfect, and Complete Binary Trees (cont.)

- A *complete binary tree* is a perfect binary tree through level *n* - 1 with some extra leaf nodes at level *n* (the tree height), all toward the left



Node, Link
Root
Branches
Successors, Children
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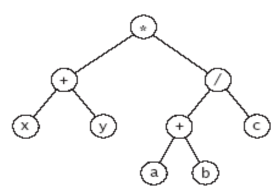
Leaf Node
Subtree
Binary Tree
Binary Search Tree
Full Binary Tree
Perfect Binary Tree
Complete Binary Tree

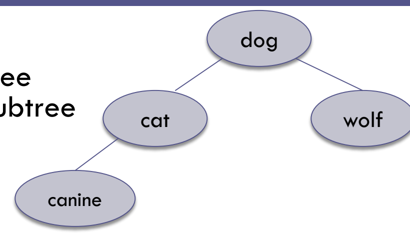
Expression Tree

- Each node contains an operator or an operand
- Operands are stored in leaf nodes
- Parentheses are not stored in the tree because the tree structure dictates the order of operand evaluation
- Operators in nodes at higher tree levels are evaluated after operators in nodes at lower tree levels



`(x + y) * ((a + b) / c)`

Binary Search Tree



- Binary search trees
  - All elements in the left subtree precede those in the right subtree
- A formal definition:

A set of nodes T is a binary search tree if either of the following is true:

- T is empty
- If T is not empty, its root node has two subtrees, T_L and T_R, such that T_L and T_R are binary search trees and the value in the root node of T is greater than all values in T_L and is less than all values in T_R

Binary Search Tree (cont.)

- A binary search tree never has to be sorted because its elements always satisfy the required order relationships
- When new elements are inserted (or removed) properly, the binary search tree maintains its order
- In contrast, a sorted array must be expanded whenever new elements are added, and compacted whenever elements are removed—expanding and contracting are both O(*n*)

Binary Search Tree (cont.)

- When searching a BST, each probe has the potential to eliminate half the elements in the tree, so searching can be O(log *n*)
- In the worst case, searching is O(*n*)

Recursive Algorithm for Searching a Binary Tree

1. **`if`** the tree is empty
2. return null *(target is not found)*

   `else if` the target matches the root node's data
3. return the data stored at the root node

   `else if` the target is less than the root node's data
4. return the result of searching the left subtree of the root

   `else`
5. return the result of searching the right subtree of the root

Tree Traversals

Section 6.2

Tree Traversals

- Often we want to determine the nodes of a tree and their relationship
  - We can do this by walking through the tree in a prescribed order and visiting the nodes as they are encountered
  - This process is called *tree traversal*
- Three common kinds of tree traversal
  - Inorder
  - Preorder
  - Postorder

Tree Traversals (cont.)

- Preorder: visit root node, traverse T_L, traverse T_R
- Inorder: traverse T_L, visit root node, traverse T_R
- Postorder: traverse T_L, traverse T_R, visit root node

Algorithm for Preorder Traversal

1. if the tree is empty
2. Return.
   else
3. Visit the root.
4. Preorder traverse the left subtree.
5. Preorder traverse the right subtree.

Algorithm for Inorder Traversal

1. if the tree is empty
2. Return.
   else
3. Inorder traverse the left subtree.
4. Visit the root.
5. Inorder traverse the right subtree.

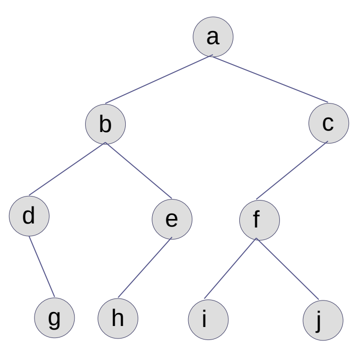
Algorithm for Postorder Traversal

1. if the tree is empty
2. Return.
   else
3. Postorder traverse the left subtree.
4. Postorder traverse the right subtree.
5. Visit the root.

Visualizing Tree Traversals

Algorithm for Preorder Traversal

1. if the tree is empty
2. Return.
   else
3. Visit the root.
4. Preorder traverse the left subtree.
5. Preorder traverse the right subtree.



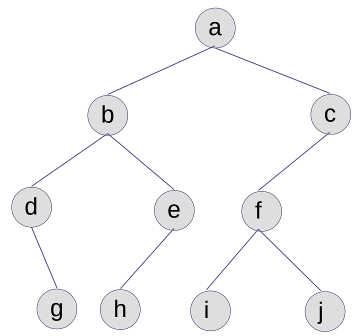
a b d g e h c f i j

Visualizing Tree Traversals

Algorithm for Inorder Traversal

1. `if` the tree is empty
2. Return.

   `else`
3. Inorder traverse the left subtree.
4. Visit the root.
5. Inorder traverse the right subtree.



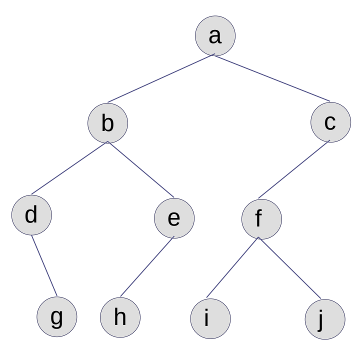
d g b h e a i f j c

Visualizing Tree Traversals

Algorithm for Postorder Traversal

1. `if` the tree is empty
2. Return.

   `else`
3. Postorder traverse the left subtree.
4. Postorder traverse the right subtree.
5. Visit the root.



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Tree Traversals (cont.)

- Preorder: visit root node, traverse T_L, traverse T_R
- Inorder: traverse T_L, visit root node, traverse T_R
- Postorder: traverse T_L, traverse T_R, visit root node

Algorithm for Preorder Traversal

1. if the tree is empty
2. Return.

else

3. Visit the root.
4. Preorder traverse the left subtree.
5. Preorder traverse the right subtree.

Algorithm for Inorder Traversal

1. if the tree is empty
2. Return.

else

3. Inorder traverse the left subtree.
4. Visit the root.
5. Inorder traverse the right subtree.

Algorithm for Postorder Traversal

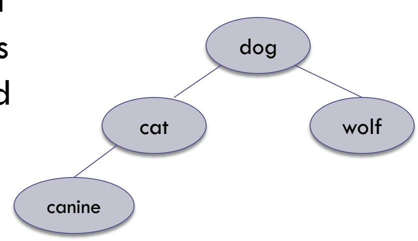
1. if the tree is empty
2. Return.

else

3. Postorder traverse the left subtree.
4. Postorder traverse the right subtree.
5. Visit the root.

Traversals of Binary Search Trees and Expression Trees

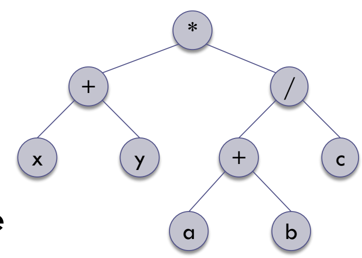
- An inorder traversal of a binary search tree results in the nodes being visited in sequence by increasing data value



canine, cat, dog, wolf

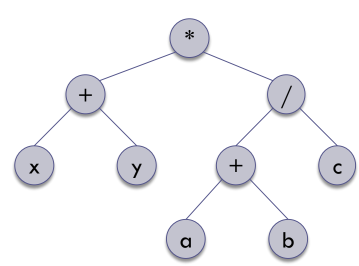
Traversals of Binary Search Trees and Expression Trees (cont.)

- An inorder traversal of an expression tree results in the sequence
  x + y * a + b / c
- If we insert parentheses where they belong, we get the infix form:
  `(x + y) * ((a + b) / c)`



Traversals of Binary Search Trees and Expression Trees (cont.)

- A postorder traversal of an expression tree results in the sequence
  x y + a b + c / *
- This is the *postfix* or *reverse polish* form of the expression
- Operators follow operands



Traversals of Binary Search Trees and Expression Trees (cont.)

- A preorder traversal of an expression tree results in the sequence
  * + x y / + a b c
- This is the *prefix* or *forward polish* form of the expression
- Operators precede operands