# SORTING

Chapter 8

## Comparison of Quadratic Sorts

2

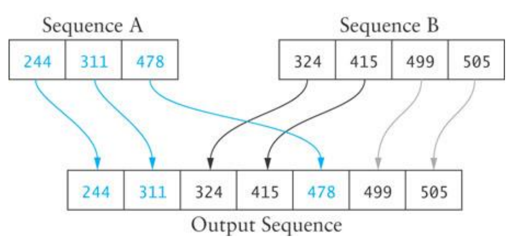
| | Number of Comparisons | | Number of Exchanges | |
|---|---|---|---|---|
| | Best | Worst | Best | Worst |
| Selection sort | $O(n^2)$ | $O(n^2)$ | $O(n)$ | $O(n)$ |
| Bubble sort | $O(n)$ | $O(n^2)$ | $O(1)$ | $O(n^2)$ |
| Insertion sort | $O(n)$ | $O(n^2)$ | $O(n)$ | $O(n^2)$ |

# Merge Sort

Section 8.7

## Merge

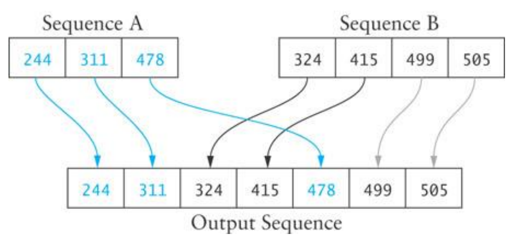
- A *merge* is a common data processing operation performed on two ordered sequences of data.
  - The result is a third ordered sequence containing all the data from the first two sequences



# Merge Algorithm

**Merge Algorithm**

1. Access the first item from both sequences.
2. `while` not finished with either sequence
3. Compare the current items from the two sequences, copy the smaller current item to the output sequence, and access the next item from the input sequence whose item was copied.
4. Copy any remaining items from the first sequence to the output sequence.
5. Copy any remaining items from the second sequence to the output sequence.




# Analysis of Merge

- For two input sequences each containing *n* elements, each element needs to move from its input sequence to the output sequence
- Merge time is O(*n*)
- Space requirements
  - The array cannot be merged in place
  - Additional space usage is O(*n*)

# Code for Merge

```
private static void merge(T[] out, T[] left, T[] right) {
   // merge left and right into out
   // Access first item from all sequences
   int i = 0; // left
   int j = 0; // right
   int k = 0; // out
   // while there is data in both left and right
   while (i < left.length && j < right.length) {
      // find smaller and insert into out
      if (left[i].compareTo(right[j]) < 0)
         out[k++] = left[i++];
      else
         out[k++] = right[j++];
   }
   // Copy remaining items from left into out
   while (i < left.length)
      out[k++] = left[i++];
   // Copy remaining items from right into out
   while (j < right.length)
      out[k++] = right[j++];
} // merge()
```

# Merge Sort

- We can modify merging to sort a single, unsorted array
  1. Split the array into two halves
  2. Sort the left half
  3. Sort the right half
  4. Merge the two

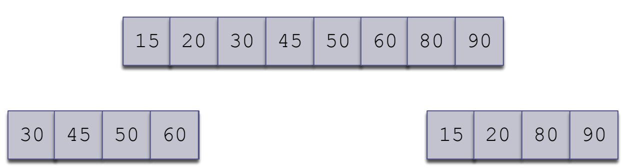
- This algorithm can be written with a recursive step

# (recursive) Algorithm for Merge Sort

**Algorithm for Merge Sort**

1. `if` the `tableSize` is > 1
2. Set `halfSize` to `tableSize` divided by 2.
3. Allocate a table called `leftTable` of size `halfSize`.
4. Allocate a table called `rightTable` of size `tableSize - halfSize`.
5. Copy the elements from `table[0 ... halfSize - 1]` into `leftTable`.
6. Copy the elements from `table[halfSize ... tableSize]` into `rightTable`.
7. Recursively apply the merge sort algorithm to `leftTable`.
8. Recursively apply the merge sort algorithm to `rightTable`.
9. Apply the merge method using `leftTable` and `rightTable` as the input and the original table as the output.

# Trace of Merge Sort (cont.)



## Analysis of Merge Sort

- Each backward step requires a movement of *n* elements from smaller-size arrays to larger arrays; the effort is O(*n*)
- The number of steps which require merging is log *n* because each recursive call splits the array in half
- The total effort to reconstruct the sorted array through merging is O(*n* log *n*)
- Requires a total of n additional storage locations.

## Code for Merge Sort

```
public static void sort(T[] table) {
   // A table with 1 element is already sorted
   if (table.length > 1) {
      // Split table into halves
      int halfSize = table.length/2;
      T[] left = new Comparable[halfSize];
      T[] right = new Comparable[table.length - halfSize];
      System.arrayCopy(table, 0, left, 0, halfSize);
      System.arrayCopy(table, halfSize, right, 0, table.length-halfSize);
      // sort the halves
      sort(left);
      sort(right);
      // merge the halves
      merge(table, left, right);
   }
} // sort()
```

# Heapsort

Section 8.8

## Heapsort

- Merge sort time is O($n$ log $n$) but still requires, temporarily, $n$ extra storage locations
- *Heapsort* does not require any additional storage
- As its name implies, heapsort uses a heap to store the array

# First Version of a Heapsort Algorithm

- When used as a priority queue, a heap maintains a smallest value at the top
- The following algorithm
  - places an array's data into a heap,
  - then removes each heap item (O(*n* log *n*)) and moves it back into the array
- This version of the algorithm requires *n* extra storage locations

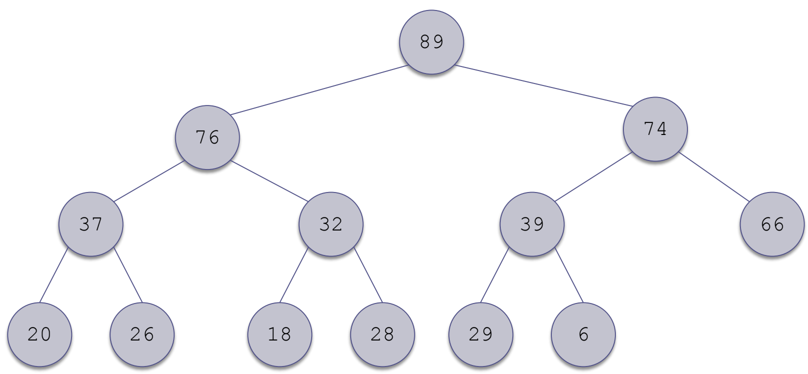
**Heapsort Algorithm: First Version**

1. **Insert each value from the array to be sorted into a priority queue (heap).**
2. **Set `i` to 0**
3. **`while` the priority queue is not empty**
4. **Remove an item from the queue and insert it back into the array at position `i`**
5. **Increment `i`**

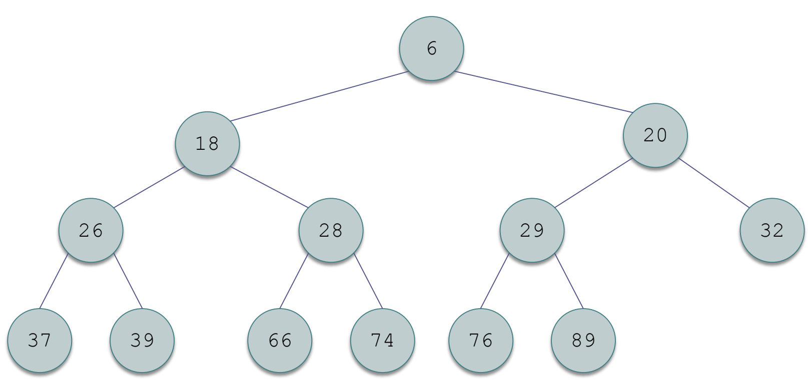
# Revising the Heapsort Algorithm

- Instead of using a Min Heap, use a Max heap
- The root contains the largest element
- Then,
  - move the root item to the bottom of the heap
  - reheap, ignoring the item moved to the bottom

# Trace of Heapsort




# Trace of Heapsort (cont.)



# Revising the Heapsort Algorithm

- If we implement the heap as an array
  - each element removed will be placed at the end of the array, and
  - the heap part of the array decreases by one element

| [0] | [1] | [2] | [3] | [4] | [5] | [6] | [7] | [8] | [9] | [10] | [11] | [12] |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 89 | 76 | 74 | 37 | 32 | 39 | 66 | 20 | 26 | 18 | 28 | 29 | 6 |

| [0] | [1] | [2] | [3] | [4] | [5] | [6] | [7] | [8] | [9] | [10] | [11] | [12] |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 76 | 37 | 74 | 26 | 32 | 39 | 66 | 20 | 6 | 18 | 28 | 29 | 89 |

| [0] | [1] | [2] | [3] | [4] | [5] | [6] | [7] | [8] | [9] | [10] | [11] | [12] |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 74 | 37 | 66 | 26 | 32 | 39 | 29 | 20 | 6 | 18 | 28 | 76 | 89 |

⋮

| [0] | [1] | [2] | [3] | [4] | [5] | [6] | [7] | [8] | [9] | [10] | [11] | [12] |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 6 | 18 | 20 | 26 | 28 | 29 | 32 | 37 | 39 | 66 | 74 | 76 | 89 |

# Algorithm for In-Place Heapsort

### Algorithm for In-Place Heapsort

1. **Build a heap by rearranging the elements in an unsorted array**
2. **`while` the heap is not empty**
3. **Remove the first item from the heap by swapping it with the last item in the heap and restoring the heap property**

# Algorithm to Build a Heap

- Start with an array `table` of length `table.length`
- Consider the first item to be a heap of one item
- Next, consider the general case where the items in array `table` from `0` through `n-1` form a heap and the items from `n` through `table.length – 1` are not in the heap

# Algorithm to Build a Heap (cont.)

**Refinement of Step 1 for In-Place Heapsort**

1.1 **`while` n is less than `table.length`**

1.2 **Increment n by 1. This inserts a new item into the heap**

1.3 **Restore the heap property**

## Analysis of Heapsort

- Because a heap is a complete binary tree, it has log *n* levels
- Building a heap of size *n* requires finding the correct location for an item in a heap with log *n* levels
- Each insert (or remove) is O(log *n*)
- With *n* items, building a heap is O(*n* log *n*)
- No extra storage is needed

## Quicksort

Section 8.9

# Quicksort

- Developed in 1962
- Quicksort selects a specific value called a pivot and rearranges the array into two parts (called *partioning*)
  - all the elements in the left subarray are less than or equal to the pivot
  - all the elements in the right subarray are larger than the pivot
  - The pivot is placed between the two subarrays
- The process is repeated until the array is sorted

# Trace of Quicksort

| | | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| 44 | 75 | 23 | 43 | 55 | 12 | 64 | 77 | 33 |

# Trace of Quicksort (cont.)

| | | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| 44 | 75 | 23 | 43 | 55 | 12 | 64 | 77 | 33 |

Arbitrarily select the first element as the pivot

# Trace of Quicksort (cont.)

| | | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| 55 | 75 | 23 | 43 | 44 | 12 | 64 | 77 | 33 |

Partition the elements so that all values less than or equal to the pivot are to the left, and all values greater than the pivot are to the right

# Trace of Quicksort (cont.)

| | | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| 12 | 33 | 23 | 43 | 44 | 55 | 64 | 77 | 75 |

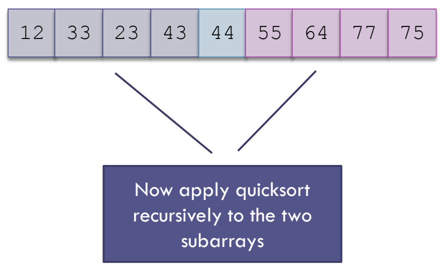
Partition the elements so that all values less than or equal to the pivot are to the left, and all values greater than the pivot are to the right

# Quicksort Example(cont.)

44 is now in its correct position

| | | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| 12 | 33 | 23 | 43 | 44 | 55 | 64 | 77 | 75 |

# Trace of Quicksort (cont.)




# Algorithm for Quicksort

- We describe how to do the partitioning later
- The indexes `first` and `last` are the end points of the array being sorted
- The index of the pivot after partitioning is `pivIndex`

**Algorithm for Quicksort**

1. **`if first < last` then**
2. **Partition the elements in the subarray `first . . . last` so that the pivot value is in its correct place (subscript `pivIndex`)**
3. **Recursively apply quicksort to the subarray `first . . . pivIndex - 1`**
4. **Recursively apply quicksort to the subarray `pivIndex + 1 . . . last`**

# Analysis of Quicksort

- If the pivot value is a random value selected from the current subarray,
  - then statistically half of the items in the subarray will be less than the pivot and half will be greater
- If both subarrays have the same number of elements (best case), there will be log *n* levels of recursion
- At each recursion level, the partitioning process involves moving every element to its correct position—*n* moves
- Quicksort is O(*n* log *n*), just like merge sort

# Analysis of Quicksort (cont.)

- The array split may not be the best case, i.e. 50-50
- An exact analysis is difficult (and beyond the scope of this class), but, the running time will be bounded by a constant x *n* log *n*

# Analysis of Quicksort (cont.)

- A quicksort will give very poor behavior if, each time the array is partitioned, a subarray is empty.
- In that case, the sort will be O($n^2$)
- Under these circumstances, the overhead of recursive calls and the extra run-time stack storage required by these calls makes this version of quicksort a poor performer relative to the quadratic sorts
  - We'll discuss a solution later

# Code for Quicksort

```
public static void sort(T[], int first, int last) {
   if (first < last) {
      // partition the table at pivotIndex
      int pivotIndex = partition(table, first, last);

      // sort the left half
      sort(table, first, pivotIndex-1);

      // sort the right half
      sort(table, pivotIndex+1, last);
   }
} // sort()
```

# Algorithm for Partitioning

| | | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| 44 | 75 | 23 | 43 | 55 | 12 | 64 | 77 | 33 |

If the array is randomly ordered, it does not matter which element is the pivot.

For simplicity we pick the element with subscript `first`

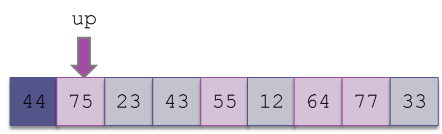
# Trace of Partitioning (cont.)

| | | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| 44 | 75 | 23 | 43 | 55 | 12 | 64 | 77 | 33 |

If the array is randomly ordered, it does not matter which element is the pivot.

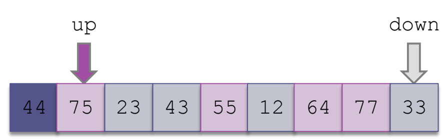
For simplicity we pick the element with subscript `first`

# Trace of Partitioning (cont.)



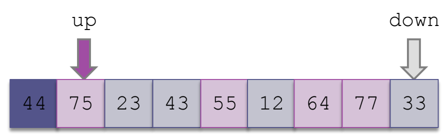

Search for the first value at the left end of the array that is greater than the pivot value

# Trace of Partitioning (cont.)



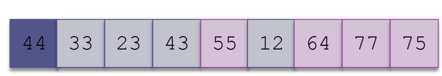

Then search for the first value at the right end of the array that is less than or equal to the pivot value

# Trace of Partitioning (cont.)




Exchange these values

# Trace of Partitioning (cont.)




Exchange these values

# Trace of Partitioning (cont.)

| | | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| 44 | 33 | 23 | 43 | 55 | 12 | 64 | 77 | 75 |

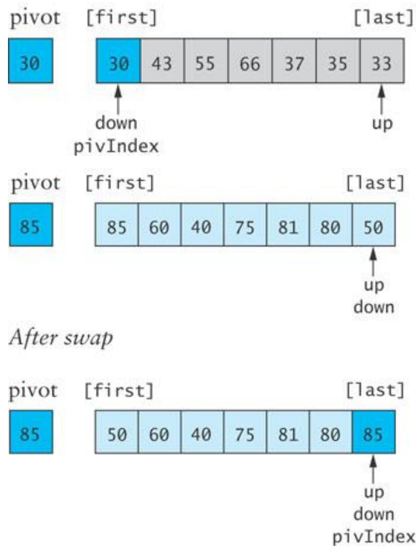
Repeat

# Algorithm for Partitioning

**Algorithm for partition Method**

1. Define the pivot value as the contents of `table[first]`.
2. Initialize `up` to `first` and `down` to `last`.
3. `do`
4. Increment `up` until `up` selects the first element greater than the pivot value or `up` has reached `last`.
5. Decrement `down` until `down` selects the first element less than or equal to the pivot value or `down` has reached `first`.
6. `if up < down` then
7. Exchange `table[up]` and `table[down]`.
8. `while up` is to the left of `down`
9. Exchange `table[first]` and `table[down]`.
10. Return the value of `down` to `pivIndex`.

## Code for `partition` when `Pivot` is the largest or smallest value




## Code for `partition` (cont.)

```
public static void partition(T[] table, int first, int last) {
   // select first element as pivot value
   // Initialize up to first and down to last
   do {
      // Increment up until it selects first element >= pivot or it reaches last
      while ((up < last) && (pivot.compareTo(table[up]) >= 0))
         up++;
      // Decrement down until it select first element < pivot or it reaches first
      while ((down > first) && (pivot.compareTo(table[down]) < 0))
         down--;
      if (up < down) {
         // exchange table[up] and table[down]
         T temp = table[up];
         table[up] = table[down];
         table[down] = temp;
      }
   while (up < down);
   // exchange table[first] and table[down]
   T temp = table[first]; table[first] = table[down]; table[down] = temp;
   // return value of down a pivotIndex
   return down;
} // partition()
```

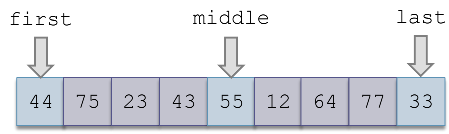
# Revised Partition Algorithm

- Quicksort is O($n^2$) when each split yields one empty subarray, which is the case when the array is presorted
- A better solution is to pick the pivot value in a way that is less likely to lead to a bad split
  - Use three references: `first, middle, last`
  - Select the median of the these items as the pivot

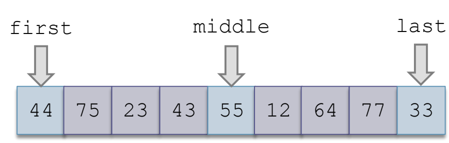
# Trace of Revised Partitioning

| | | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| 44 | 75 | 23 | 43 | 55 | 12 | 64 | 77 | 33 |

# Trace of Revised Partitioning (cont.)

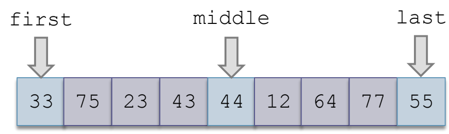



# Trace of Revised Partitioning (cont.)



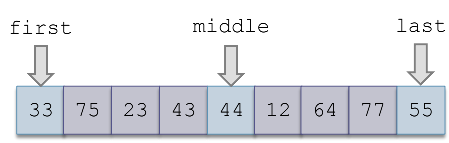

Sort these values

# Trace of Revised Partitioning (cont.)




Sort these values

Trace of Revised Partitioning (cont.)




Exchange middle with first

# Trace of Revised Partitioning (cont.)

| first | | | | middle | | | | last |
|---|---|---|---|---|---|---|---|---|
| 44 | 75 | 23 | 43 | 33 | 12 | 64 | 77 | 55 |

Exchange middle with first

# Trace of Revised Partitioning (cont.)

| | | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| 44 | 75 | 23 | 43 | 33 | 12 | 64 | 77 | 55 |

Run the partition algorithm using the first element as the pivot

# Algorithm for Revised `partition` Method

Algorithm for Revised partition Method

1. Sort `table[first]`, `table[middle]`, and `table[last]`
2. Move the median value to `table[first]` (the pivot value) by exchanging `table[first]` and `table[middle]`.
3. Initialize `up` to `first` and `down` to `last`
4. `do`
5. Increment `up` until `up` selects the first element greater than the pivot value or `up` has reached `last`
6. Decrement `down` until `down` selects the first element less than or equal to the pivot value or `down` has reached `first`
7. `if up < down` then
8. Exchange `table[up]` and `table[down]`
9. `while up` is to the left of `down`
10. Exchange `table[first]` and `table[down]`
11. Return the value of `down` to `pivIndex`

# Comparison of Sort Algorithms

Summary

# Sort Review

| Number of Comparisons | | | |
|---|---|---|---|
| | Best | Average | Worst |
| Selection sort | $O(n^2)$ | $O(n^2)$ | $O(n^2)$ |
| Bubble sort | $O(n)$ | $O(n^2)$ | $O(n^2)$ |
| Insertion sort | $O(n)$ | $O(n^2)$ | $O(n^2)$ |
| Shell sort | $O(n^{7/6})$ | $O(n^{5/4})$ | $O(n^2)$ |
| Merge sort | $O(n \log n)$ | $O(n \log n)$ | $O(n \log n)$ |
| Heapsort | $O(n \log n)$ | $O(n \log n)$ | $O(n \log n)$ |
| Quicksort | $O(n \log n)$ | $O(n \log n)$ | $O(n^2)$ |