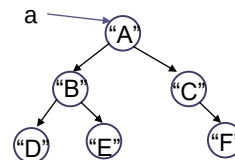


# IMPLEMENTING BINARY TREES

## Chapter 6

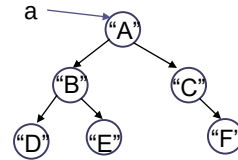
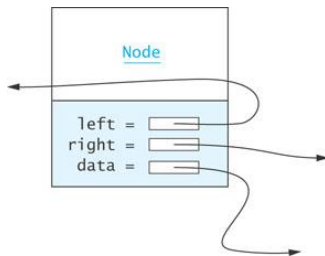
### A Binary Tree

```
BinaryTree<String> a = new BinaryTree<String>();
```



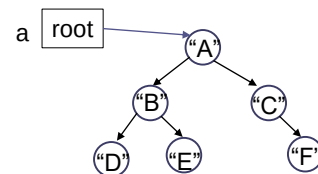
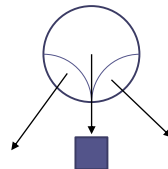
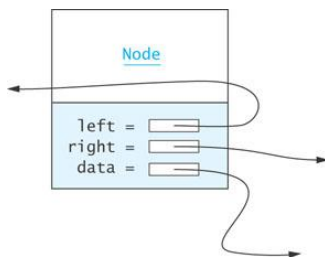
# A Binary Tree

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BinaryTree<String> a = new BinaryTree<String>();
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# A Binary Tree

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BinaryTree<String> a = new BinaryTree<String>();
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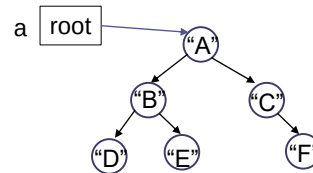
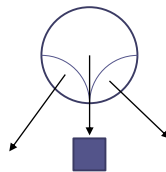


# A Binary Tree

```
BinaryTree<String> a = new BinaryTree<String>();
```

An Empty Tree

a root = null



## Implementing a `BinaryTree` Class

### Section 6.3

## BinaryTree<E> Class (cont.)

```
public class BinaryTree<E> {
    // Data members/fields...just the root is needed
    - Node<E> root;

    // Constructor(s)
    + BinaryTree()
    + BinaryTree(Node<E> node)
    + BinaryTree(E data, BinaryTree<E> leftTree, BinaryTree<E> rightTree)

    // Methods: Accessors
    + E getData() throws BinaryTreeException
    + boolean isEmpty()
    + void clear()
    + BinaryTree<E> getLeftSubtree()
    + BinaryTree<E> getRightSubtree()
    + boolean isLeaf()
    + int height()
    - int height(Node<E> node)
    + String toString()
    - void preOrderTraverse(Node<E> node, int depth, StringBuilder sb) {
} // BinaryTree<E> class
```

## BinaryTree<E> Class (cont.)

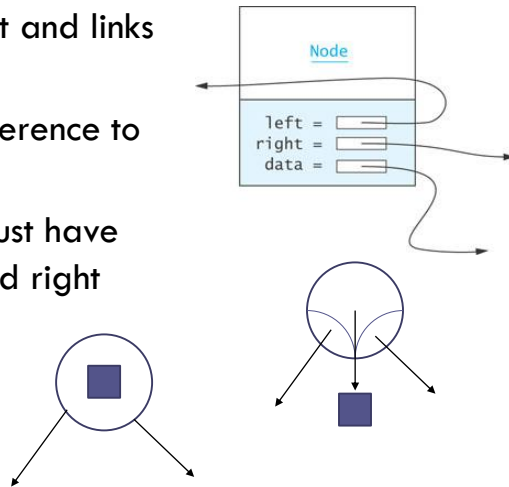
```
public class BinaryTree<E> {
    // Data members/fields...just the root is needed
    - Node<E> root;

    // Constructor(s)
    + BinaryTree()
    + BinaryTree(Node<E> node)
    + BinaryTree(E data, BinaryTree<E> leftTree, BinaryTree<E> rightTree)

    // Methods: Accessors
    + E getData() throws BinaryTreeException
    + boolean isEmpty()
    + void clear()
    + BinaryTree<E> getLeftSubtree()
    + BinaryTree<E> getRightSubtree()
    + boolean isLeaf()
    + int height()
    - int height(Node<E> node)
    + String toString()
    - void preOrderTraverse(Node<E> node, int depth, StringBuilder sb) {
} // BinaryTree<E> class
```

## Node<E> Class

- Just as for a linked list, a node consists of a data part and links to successor nodes
- The data part is a reference to type E
- A binary tree node must have links to both its left and right subtrees

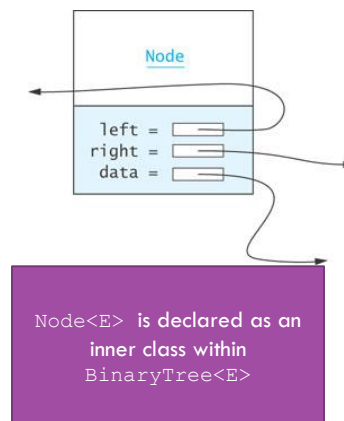


## Node<E> Class (cont.)

```
public class Node<E> {
    public E data;
    public Node<E> left;
    public Node<E> right;

    public Node(E data) {
        this.data = data;
        left = null;
        right = null;
    } // Node()

    public String toString() {
        return data.toString();
    } // toString()
} class Node<E>
```



## BinaryTree<E> Class (cont.)

```
public class BinaryTree<E> {
    // Data members/fields...just the root is needed
    - Node<E> root;

    // Constructor(s)
    + BinaryTree()
    + BinaryTree(Node<E> node)
    + BinaryTree(E data, BinaryTree<E> leftTree, BinaryTree<E> rightTree)

    // Methods: Accessors
    + E getData() throws BinaryTreeException
    + boolean isEmpty()
    + void clear()
    + BinaryTree<E> getLeftSubtree()
    + BinaryTree<E> getRightSubtree()
    + boolean isLeaf()
    + int height()
    - int height(Node<E> node)
    + String toString()
    - void preOrderTraverse(Node<E> node, int depth, StringBuilder sb) {
} // BinaryTree<E> class
```

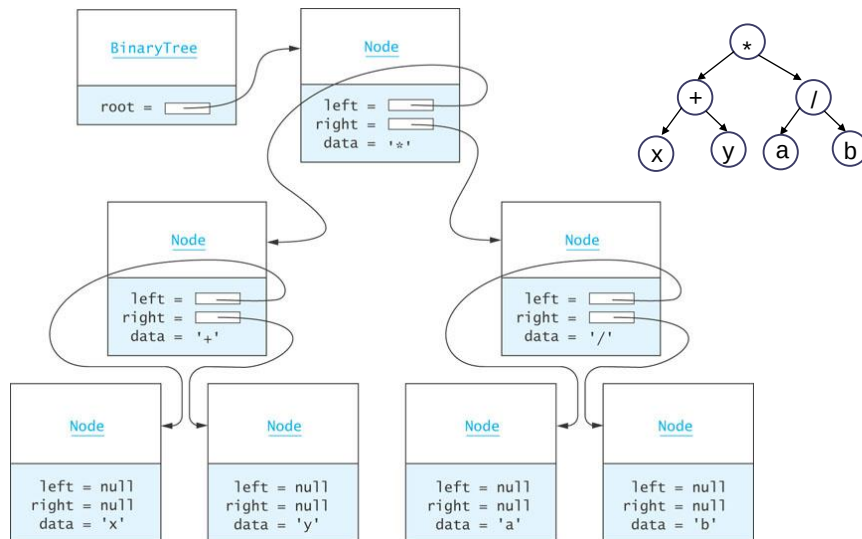
## BinaryTree<E> Class (cont.)

```
public class BinaryTree<E> {
    // Data members/fields...just the root is needed
    - Node<E> root;

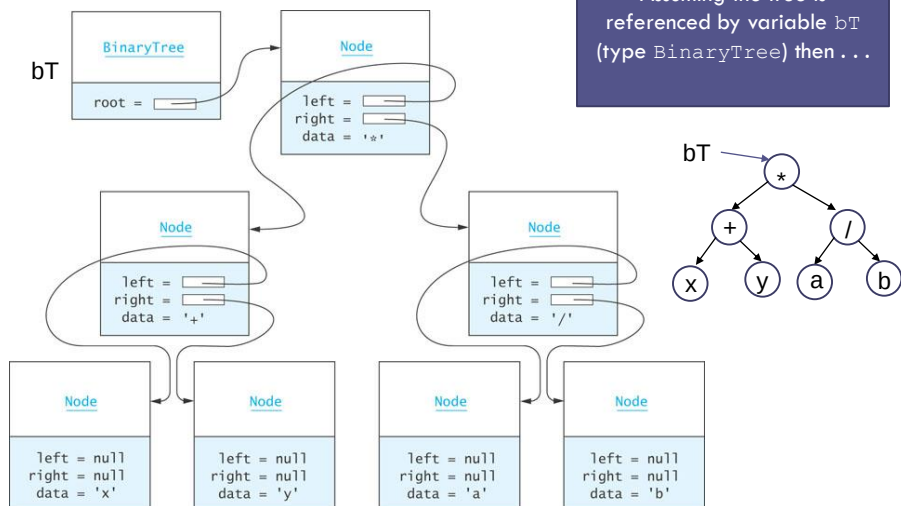
    // Constructor(s)
    + BinaryTree()
    + BinaryTree(Node<E> node)
    + BinaryTree(E data, BinaryTree<E> leftTree, BinaryTree<E> rightTree)

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    + E getData() throws BinaryTreeException
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    + int height()
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    + String toString()
    - void preOrderTraverse(Node<E> node, int depth, StringBuilder sb) {
} // BinaryTree<E> class
```

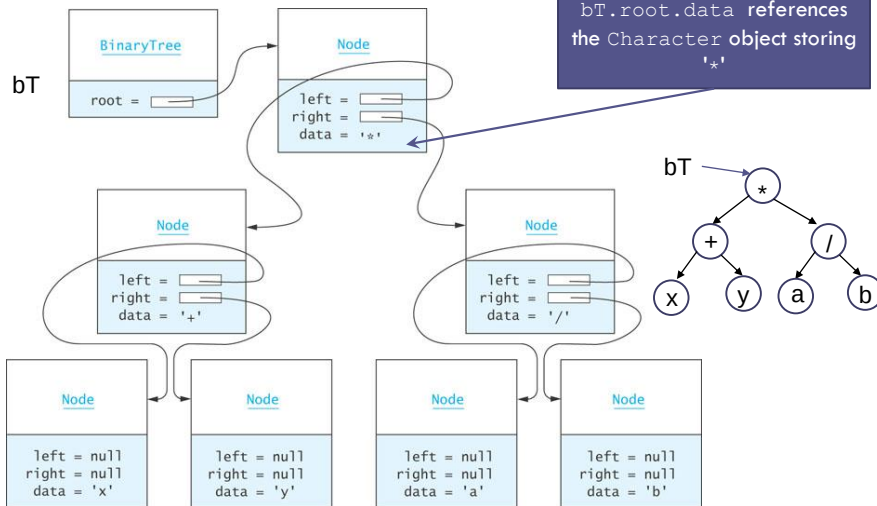
## BinaryTree<E> Class (cont.)



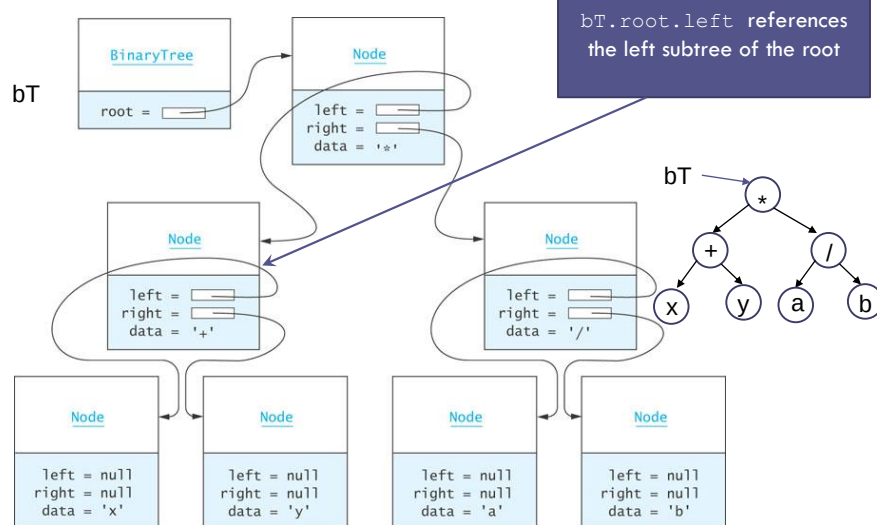
## BinaryTree<E> Class (cont.)



## BinaryTree<E> Class (cont.)

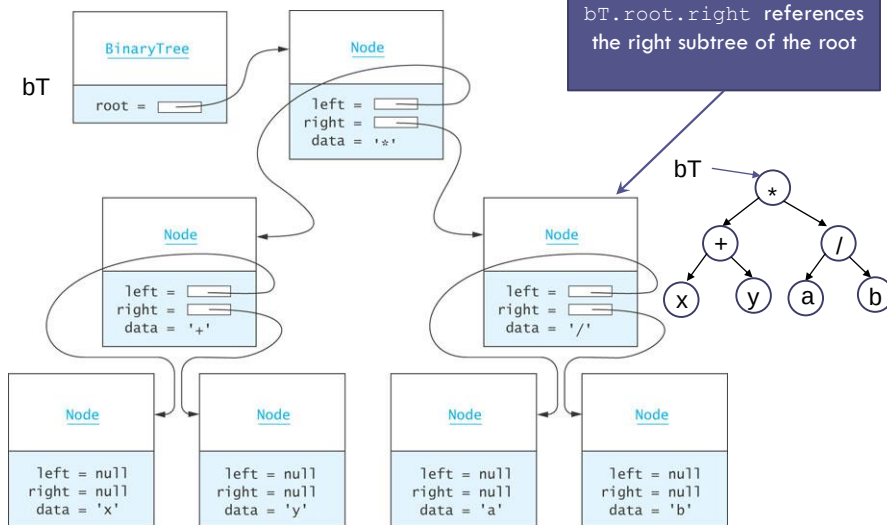


## BinaryTree<E> Class (cont.)

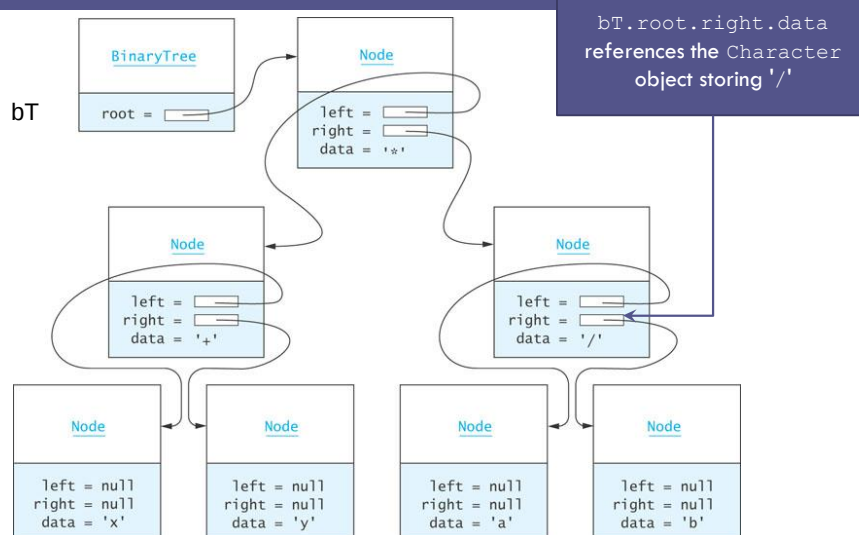




## BinaryTree<E> Class (cont.)



## BinaryTree<E> Class (cont.)



## BinartTree<E> Class

```
// Constructors
public BinaryTree() {
    root = null;
} // BinaryTree()

public BinaryTree(Node<E> root) {
    this.root = root;
} // BinaryTree()
```

## BinartTree<E> Class

```
// Constructors
public BinaryTree(E data, BinaryTree<E> leftTree,
    BinaryTree<E> rightTree) {

    root = new Node<E>(data);
    if (leftTree != null) {
        root.left = leftTree.root;
    } else {
        root.left = null;
    }
    if (rightTree != null) {
        root.right = rightTree.root;
    } else {
        root.right = null;
    }
} // BinaryTree()
```

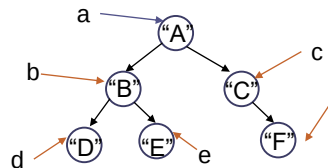
## Using BinaryTree<E> Class

```
public static void main(String[] args) {
    // Create some Binary trees to put together
    // a tree [a [b [d][e]], [c [][f]]]

    BinaryTree<String> d = new BinaryTree<String>("D", null, null);
    BinaryTree<String> e = new BinaryTree<String>("E", null, null);
    BinaryTree<String> f = new BinaryTree<String>("F", null, null);

    BinaryTree<String> b = new BinaryTree<String>("B", d, e);
    BinaryTree<String> c = new BinaryTree<String>("C", null, f);
    BinaryTree<String> a = new BinaryTree<String>("A", b, c);

    System.out.println(a);
} // main()
```



## BinaryTree<E> Class (cont.)

```
public class BinaryTree<E> {
    // Data members/fields...just the root is needed
    - Node<E> root;

    // Constructor(s)
    + BinaryTree()
    + BinaryTree(Node<E> node)
    + BinaryTree(E data, BinaryTree<E> leftTree, BinaryTree<E> rightTree)

    // Methods: Accessors
    + E getData() throws BinaryTreeException
    + boolean isEmpty()
    + void clear()
    + BinaryTree<E> getLeftSubtree()
    + BinaryTree<E> getRightSubtree()
    + boolean isLeaf()
    + int height()
    - int height(Node<E> node)
    + String toString()
    - void preOrderTraverse(Node<E> node, int depth, StringBuilder sb) {
} // BinaryTree<E> class
```

## BinaryTree<E> Class (cont.)

```
public class BinaryTree<E> {
    // Data members/fields...just the root is needed
    - Node<E> root;

    // Constructor(s)
    + BinaryTree()
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    + int height()
    - int height(Node<E> node)
    + String toString()
    - void preOrderTraverse(Node<E> node, int depth, StringBuilder sb) {
} // BinaryTree<E> class
```

## BinartTree<E> Class

```
// Constructors

public boolean isEmpty() {
    return this.root==null;
} // isEmpty()

public void clear() {
    this.root = null;
} // clear()
```

## BinaryTree<E> Class (cont.)

```
public class BinaryTree<E> {
    // Data members/fields...just the root is needed
    - Node<E> root;

    // Constructor(s)
    + BinaryTree()
    + BinaryTree(Node<E> node)
    + BinaryTree(E data, BinaryTree<E> leftTree, BinaryTree<E> rightTree)

    // Methods: Accessors
    + E getData() throws BinaryTreeException
    + boolean isEmpty()
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    + BinaryTree<E> getLeftSubtree()
    + BinaryTree<E> getRightSubtree()
    + boolean isLeaf()
    + int height()
    - int height(Node<E> node)
    + String toString()
    - void preOrderTraverse(Node<E> node, int depth, StringBuilder sb) {
} // BinaryTree<E> class
```

## BinaryTree<E> Class (cont.)

```
public class BinaryTreeException extends Exception {

    BinaryTreeException(String message) {
        super(message);
    } // BinaryTreeException()

} // class BinaryTreeException

public E getData() {
    if (this.root == null)
        throw new BinaryTreeException("Accessing data in null tree");

    return this.root.data;
} // getData()
```

## BinaryTree<E> Class (cont.)

```
public class BinaryTree<E> {
    // Data members/fields...just the root is needed
    - Node<E> root;

    // Constructor(s)
    + BinaryTree()
    + BinaryTree(Node<E> node)
    + BinaryTree(E data, BinaryTree<E> leftTree, BinaryTree<E> rightTree)

    // Methods: Accessors
    + E getData() throws BinaryTreeException
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    + boolean isLeaf()
    + int height()
    - int height(Node<E> node)
    + String toString()
    - void preOrderTraverse(Node<E> node, int depth, StringBuilder sb) {
} // BinaryTree<E> class
```

## BinaryTree<E> Class

```
public BinaryTree<E> getLeftSubTree() {
    if (root != null && root.left != null)
        return new BinaryTree<E>(root.left);
    else
        return new BinaryTree<E>();
} // getLeftSubTree()

public boolean isLeaf() {
    return (root == null) ||
        (root.left==null && root.right==null);
} // isLeaf()
```

## BinaryTree<E> Class (cont.)

```
public class BinaryTree<E> {
    // Data members/fields...just the root is needed
    - Node<E> root;

    // Constructor(s)
    + BinaryTree()
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    + BinaryTree(E data, BinaryTree<E> leftTree, BinaryTree<E> rightTree)

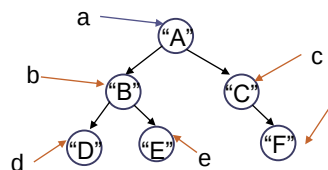
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    - void preOrderTraverse(Node<E> node, int depth, StringBuilder sb)
} // BinaryTree<E> class
```

## Using BinaryTree<E> Class

```
public static void main(String[] args) {
    // Create some Binary trees to put together
    // a tree [a [b [d][e]], [c [][f]]]

    BinaryTree<String> d = new BinaryTree<String>("D", null, null);
    BinaryTree<String> e = new BinaryTree<String>("E", null, null);
    BinaryTree<String> f = new BinaryTree<String>("F", null, null);
    BinaryTree<String> b = new BinaryTree<String>("B", d, e);
    BinaryTree<String> c = new BinaryTree<String>("C", null, f);
    BinaryTree<String> a = new BinaryTree<String>("A", b, c);

    System.out.println(a);
} // main()
```

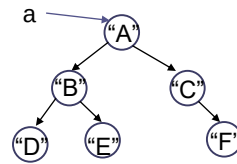


## What to print?

```

A
  B
    D
      null
      null
    E
      null
      null
  C
    null
    F
      null
      null

```



## BinaryTree<E> Class

```

public String toString() {
    StringBuilder sb = new StringBuilder();
    preOrderTraverse(this.root, 1, sb);
    return sb;
} // toString()

```



## BinaryTree<E> Class

```

public String toString() {
    StringBuilder sb = new StringBuilder();
    preOrderTraverse(this.root, 1, sb);
    return sb;
} // toString()

private void preOrderTraverse(Node<E> node, int depth,
                               StringBuilder sb) {
    for (int i=0; i < depth; i++)
        sb.append("  ");

    if (node == null)
        sb.append("null\n");
    else {
        sb.append(node.toString());
        sb.append("\n");
        preOrderTraverse(node.left, depth+1, sb);
        preOrderTraverse(node.right, depth+1, sb);
    }
} // preOrderTraverse()

```

## BinaryTree<E> Class (cont.)

```

public class BinaryTree<E> {
    // Data members/fields...just the root is needed
    - Node<E> root;

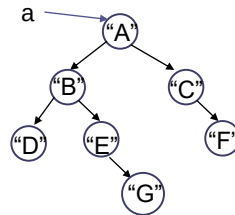
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    + boolean isLeaf()
    + int height()
    - int height(Node<E> node)
    + String toString()
    - void preOrderTraverse(Node<E> node, int depth, StringBuilder sb)
} // BinaryTree<E> class

```

## Height of a Binary Tree

$$\text{height}(\text{tree}) = \begin{cases} 0 & \text{if tree is empty} \\ 1 + \max(\text{height}(\text{leftSubtree}), \text{height}(\text{rightSubtree})) & \end{cases}$$



## Height of a Binary Tree

$$\text{height}(\text{tree}) = \begin{cases} 0 & \text{if tree is empty} \\ 1 + \max(\text{height}(\text{leftSubtree}), \text{height}(\text{rightSubtree})) & \end{cases}$$

```

public int height() {
    return height(root);
} // height()

private int height(Node<e> node) {
    if (node == null)
        return 0;
    else
        return 1 + Math.max(height(node.left), height(node.right));
} // height()

```

## BinaryTree<E> Class (cont.)

```
public class BinaryTree<E> {
    // Data members/fields...just the root is needed
    - Node<E> root;

    // Constructor(s)
    + BinaryTree()
    + BinaryTree(Node<E> node)
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```