Trees

Bryn Mawr College
CS206 Intro to Data Structures

Tree

- A tree consists of a set of nodes and a set of edges that connect pairs of nodes.
- Property: there is exactly one path (no more, no less) between any two nodes of the tree.
- A path is a connected sequence of zero or more edges.
- In a rooted tree, one distinguished node is called the root. Every node c, except the root, has exactly one parent node p, which is the first node traversed on the path from c to the root. c is p's child.
- The root has no parent.
- A node can have any number of children.

Rooted Tree Terminology

- A leaf is a node with no children.
- Siblings are nodes with the same parent.
- The ancestors of a node d are the nodes on the path from d to the root. These include d's parent, d's parent's parent, d's parent's parent's parent, and so forth up to the root. Note that d's ancestors include d itself. The root is an ancestor of every node in the tree.
- If a is an ancestor of d, then d is a descendant of a.
- The length of a path is the number of edges in the path.
- The depth of a node n is the length of the path from n to the root. (The depth of the root is zero.)

Rooted Tree Terminology (cont.)

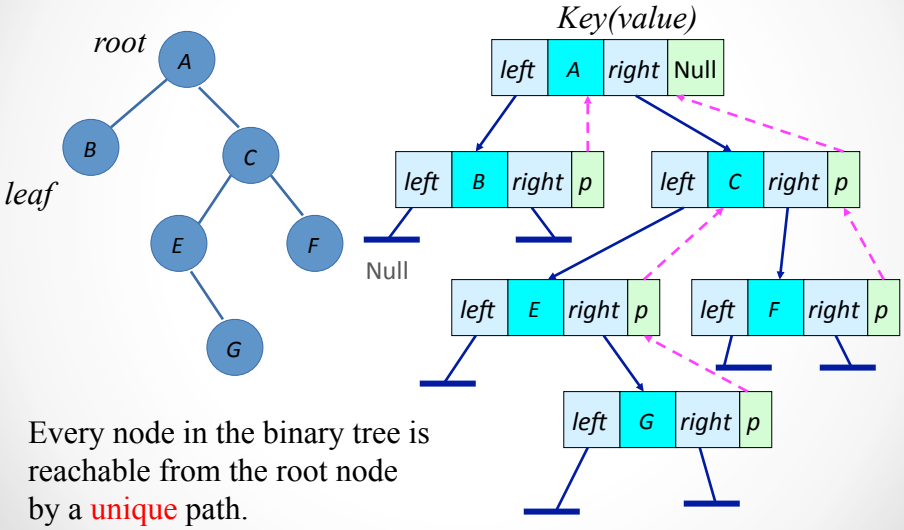
- The height of a node n is the length of the path from n to its deepest descendant. (The height of a leaf is zero.)
- The height of a tree is the depth of its deepest node = height of the root.
- The subtree rooted at node n is the tree formed by n and its descendants.
- A binary tree is a tree in which no node has more than two children, and every child is either a left child or a right child, even if it is the only child its parent has.

Binary Trees

Rooted trees can also be defined recursively. Here is the definition of a binary tree:

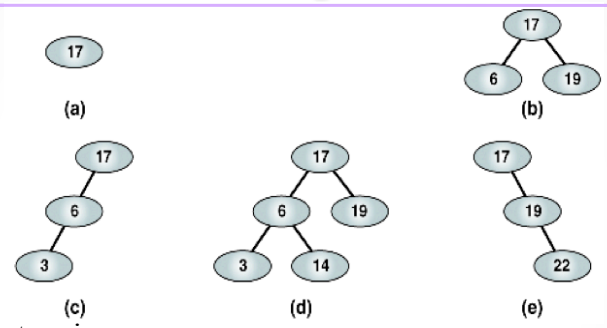
- A binary tree T is a structure defined on a finite set of nodes that either
  - Contains no nodes, or
  - Is composed of three disjoint sets of nodes:
    - a root node,
    - a binary tree called the left subtree of T, and
    - a binary tree called the right subtree of T.

A Binary Tree



Every node in the binary tree is reachable from the root node by a unique path.

Examples



- A binary tree is
  - full if every node other than leaves has two children; (a), (b), (d)
  - complete if every level is completely filled; (a), (b)
  - nearly complete if every level except the last is completely filled, and all nodes are as far left as possible; (d)
  - balanced if the depth of left and right subtrees of every node differ at most 1. (a), (b), (d)