Priority Queues

cs206

April 16

Priority Queue

- A queue that maintains order of elements according to some priority
  - Removal order, not general order
    - the rest may or may not be sorted

Key Value Pairs

- Priority Queues are usually described as being on Key-Value Pairs
    - akin to Hashtables
- Priority queues are ordered by the key, which may be:
  - derived from the Value element (which may be a large, complex object)
    - one field
    - combination of fields
  - independent of Value element
    - for example: insertion time
- best practice is make keys implement `Comparable` relation between keys using `compareTo`
- Keys ideally:
  - are unique
    - how to handle duplicate keys?
  - have a natural ordering.
    - Contrast to hashtables in which key ordering is irrelevant

Priority Queues in real world

- Homework
  - key= f(due date, difficulty, annoyance)
- Others items in priority queues
  - what is the key?

PriorityQueue Interface

```
public interface QueueInterface<Q> {
    boolean isEmpty();
    int size();
    boolean offer(Q q);
    Q poll();
    Q peek();
}
```

```
public interface PriorityQInterface<K extends Comparable<K>, V> {
    boolean isEmpty();
    int size();
    boolean offer(K key, V value);
    V poll();
    V peek();
}
```

AbstractPriorityQueue

```
public abstract class AbstractPriorityQueue <K extends Comparable<K>, V> implements PriorityQInterface<K,V> {

    protected class Pair<L extends Comparable<L>, W> implements Comparable<Pair<L,W>> {
        /** Hold the key */
        final L theK;
        /** Hold the  value*/
        final W theV;
        /**
         * Create an Entry instance
         * @param kk the key
         * @param vv the value
         */
        public Pair(L kk, W vv) {
            theK = kk;
            theV = vv;
        }
        @Override
        public int compareTo(AbstractPriorityQueue<K, V>.Pair<L, W> o) {
            return theK.compareTo(o.theK);
        }

        public String toString() {
            return "{{"+theK+" " +theV+"}}";
        }

    }
```

PQ Implementation

- Questions:
  - How to store keys and values
    - handling of duplicate keys
  - Is the storage:
    - ordered?
    - size bound?

(Internally Unordered) Priority Q

```
public class PriorityQueue<K extends Comparable<K>, V>  extends AbstractPriorityQueue<K,V> {
    /** Default capacity */
    private static int CAPACITY = 200;
    private Pair<K,V>[] pqStore;
    /** The number of items in the priority queue */
    private int size;
    public PriorityQueue() {
        this(CAPACITY);
    }
    @SuppressWarnings("unchecked")
    /**
     * Return an array list of the given capacity
     * @param initialCapacity -- the capacity
     */
    public PriorityQueue(int initialCapacity) {
        pqStore = (Pair<K,V>[]) new Pair[initialCapacity];
        this.size=0;
    }
    public int size() {
        return size;
    }
    public boolean isEmpty() {
        return size==0;
    }
    public boolean offer(K newK, V newV) {
        if (size==CAPACITY)
            return false;
        Pair<K,V> entry = new Pair<>(newK, newV);
        pqStore[size]=entry;
        size++;
        return true;
    }
```

peek & poll

```
public V peek() {
    if (isEmpty())
        return null;
    int lmin = getNext();
    Pair<K,V> entry = pqStore[lmin];
    return entry.theV;
}
```

```
public V poll() {
    if (isEmpty())
        return null;
    int lmin = getNext();
    Pair<K,V> entry = pqStore[lmin];
    remove(lmin);
    return entry.theV;
}
```

getNext(), remove(lmin)

write them.

Example

```
PriorityQueue<Integer, String> pq = new PriorityQueue<>(Ordering.MIN);
        pq.offer(1,"Jane");
        pq.offer(10,"WET");
        pq.offer(5, "WAS");
        System.out.println(pq.poll());
        System.out.println(pq.poll());
        System.out.println(pq.poll());
        System.out.println();

        pq = new PriorityQueue<>(Ordering.MAX);
        pq.offer(1,"Jane");
        pq.offer(10,"WET");
        pq.offer(5, "WAS");
        System.out.println(pq.poll());
        System.out.println(pq.poll());
        System.out.println(pq.poll());
```

(Internally Ordered) Priority Q

```
public class PriorityQueueSAL<K extends Comparable<K>, V> extends AbstractPriorityQueue<K,V> {
   final private SAL<Pair<K,V>> pqStore;
   public PriorityQueueSAL() { this(Ordering.ASCENDING); }
    public PriorityQueueSAL(Ordering order) {
        this.order=order;
        pqStore = new SAL<>(SAL.Ordering.DESCENDING);
    }
   public int size() {
        return pqStore.size();
    }
   public boolean isEmpty() {
        return pqStore.isEmpty();
   }
    public boolean offer(K newK, V newV) {
        pqStore.add(new Pair<>(newK, newV));
               return true;  // Note that this always succeeds, so always return true.
    }
   public V poll() {
        if (isEmpty())
            return null;
        Pair<K,V> p = pqStore.getAndRemove(pqStore.size()-1);
        return p.theV;
    }
```

Complexity Analysis

	Unordered	Ordered (using SAL)	Heap Based
offer			
peek			
poll			

Unordered PQ == Selection Sort

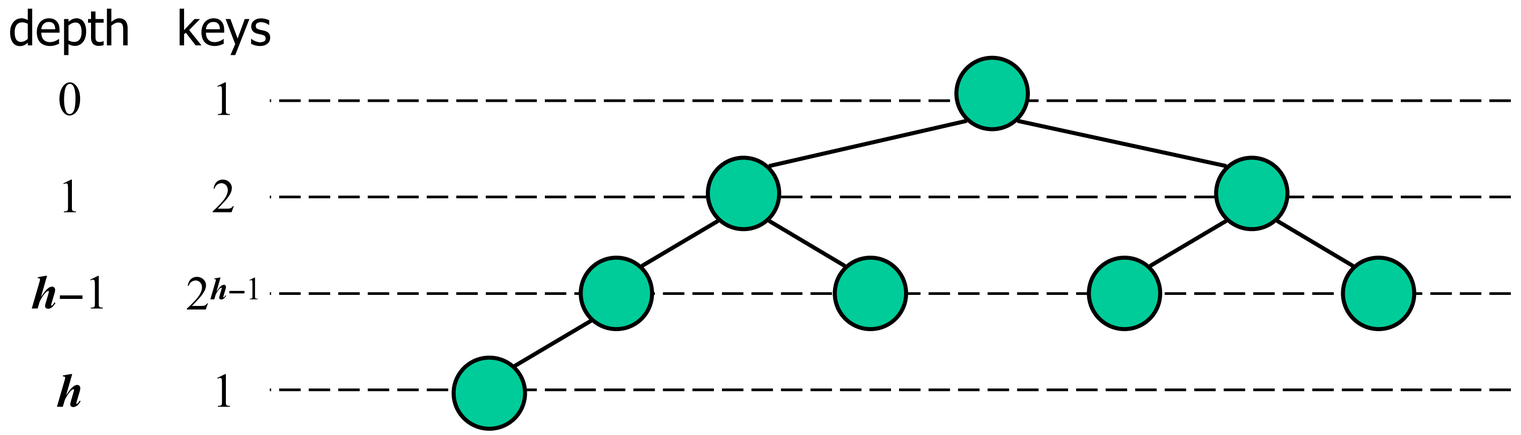
Ordered PQ = Insertion Sort

Binary Heap

- A heap is a “binary tree” storing keys at its nodes and satisfying:
  - heap-order: for every internal node v other than root, $key(v) \geq key(parent(v))$
  - Heap is filled from top down and within a level from left to right.
    - at depth h, the leaf nodes are in the leftmost positions
    - last node of a heap is the rightmost node of max depth

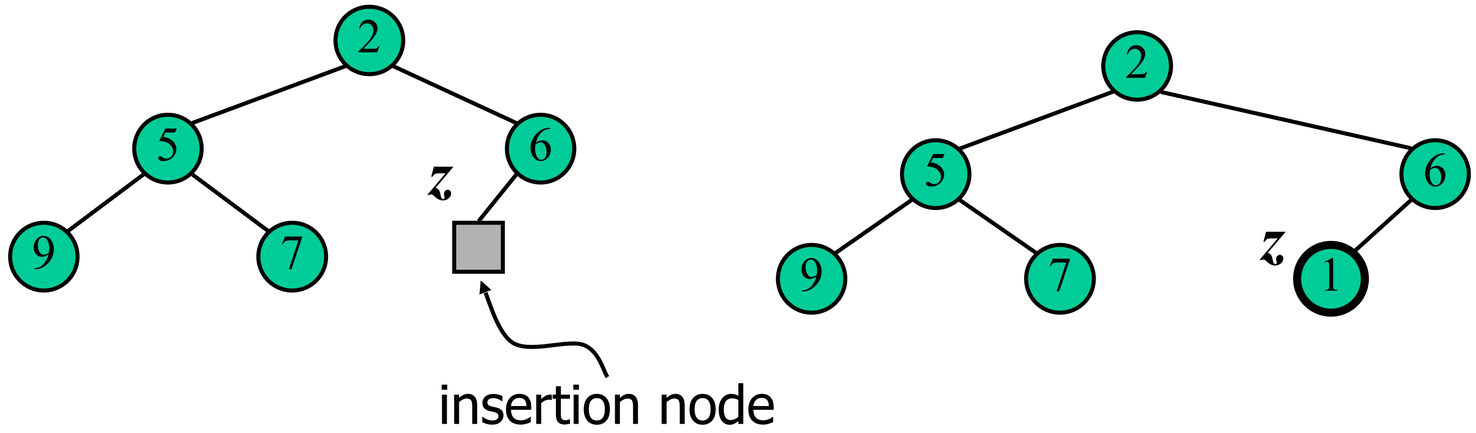
Height of a Heap

- A binary heap storing n keys has a height of $O(\log_2 n)$



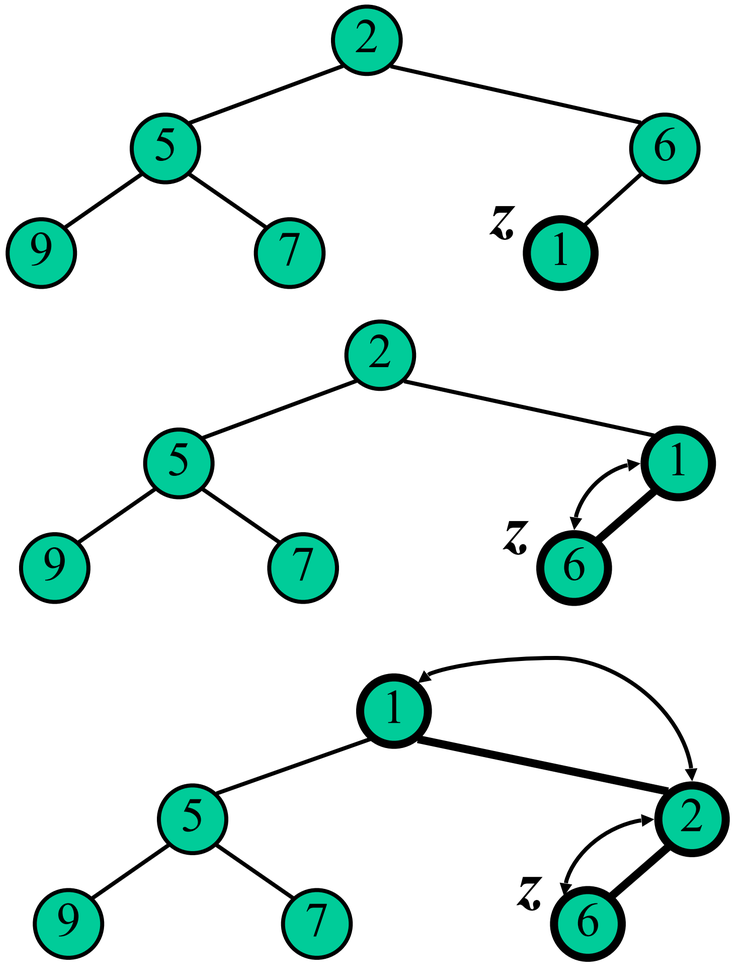
Insertion into a Heap

- Insert as new last node
- Need to restore heap order



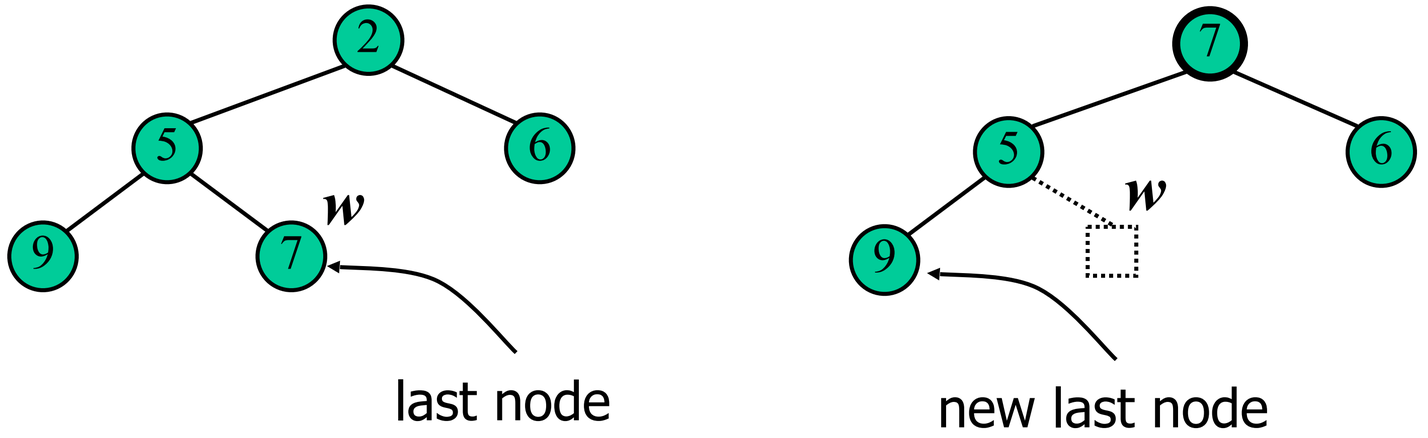
Upheap

- Restore heap order
  - swap upwards
  - stop when finding a smaller parent
  - or reach root
- $O(logn)$



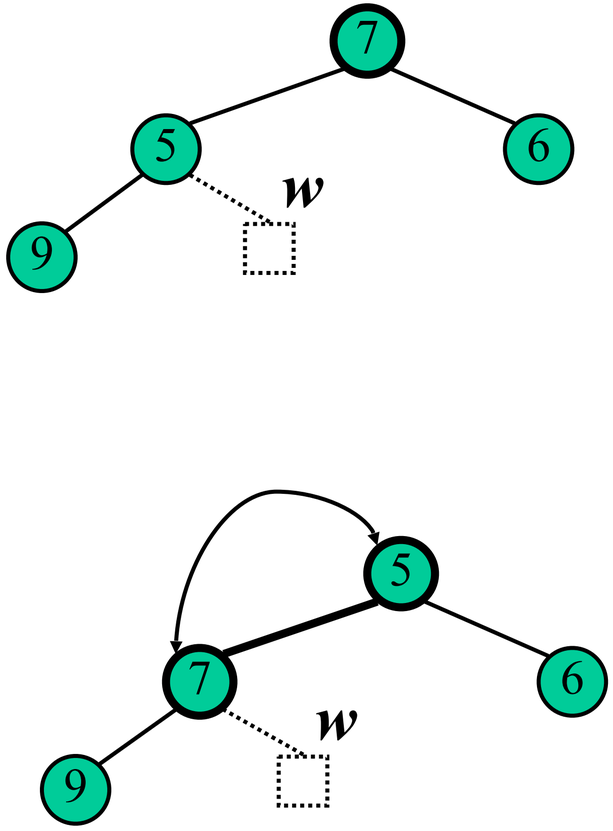
Poll

- Removing the root of the heap
  - Replace root with last node
  - Remove last node
  - Restore heap order

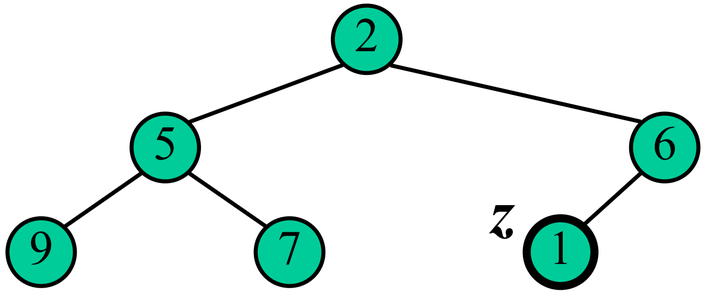


Downheap

- Restore heap order
  - swap downwards
  - swap with smaller child
  - stop when finding larger children
  - or reach a leaf
- $O(logn)$



Heaps are built on Arrays



Add to Heap here

0	1	2	3	4	5	6	7
2	5	6	9	7	1		

Locations of Parents and children are in strict mathematical relationship

- Parent from child
  - suppose child is at location childLoc in array
    - parentLoc = (childLoc-1)/2
- Child from Parent
  - suppose parent is at parentLoc in array
    - leftChild = parentLoc*2+1
    - rightChild = parentLoc*2+2

- Parent from child
  - child at loc 4 (value 7)
  - parent is at (4-1)/2 = 1 (value 5)
- Child from Parent
  - parent at loc 2 (value 6)
    - leftChild =2*2+1 = 5 (value 1)
    - rightChild = 2*2+2 = 6 (value — not used)

Priority Queue using Heaps

startup

```
public class PriorityQHeap<K extends Comparable<K>, V> extends AbstractPriorityQueue<K, V>
{

   private static final int CAPACITY = 1032;
   private Pair<K,V>[] backArray;
   private int size;

    public PriorityQHeap() {
        this(CAPACITY);
    }

    public PriorityQHeap(int capacity) {
        size=0;
        backArray = new Pair[capacity];
    }
    @Override
    public int size()
    {
        return size;
    }

    @Override
    public boolean isEmpty()
    {
        return size==0;
    }
```

Heap Insertion

Priority Queue offer method

```
public boolean offer(K key, V value)
    {
        if (size>=(backArray.length-1))
            return false;
    // put new item in at end data items
    int loc = size++;
    backArray[loc] = new Pair<K,V>(key, value);
    // up heap
    int upp = (loc-1)/2; //the location of the parent
    while (loc!=0) {
        if (0 > backArray[loc].compareTo(backArray[upp])) {
            // swap and climb
            Pair<K,V> tmp = backArray[upp];
            backArray[upp] = backArray[loc];
            backArray[loc] = tmp;
            loc = upp;
            upp = (loc-1)/2;
            }
        else
        {
            break;
        }
    }
    return true;
    }
```

Peek and Poll

```
@Override
public V poll() {
    if (isEmpty())
        return null;
    Entry<K,V> tmp = backArray[0];
    removeTop();
    return tmp.theV;
}

@Override
public V peek() {
    if (isEmpty())
        return null;
    return backArray[0].theV;
}
```

Remove head item from Heap

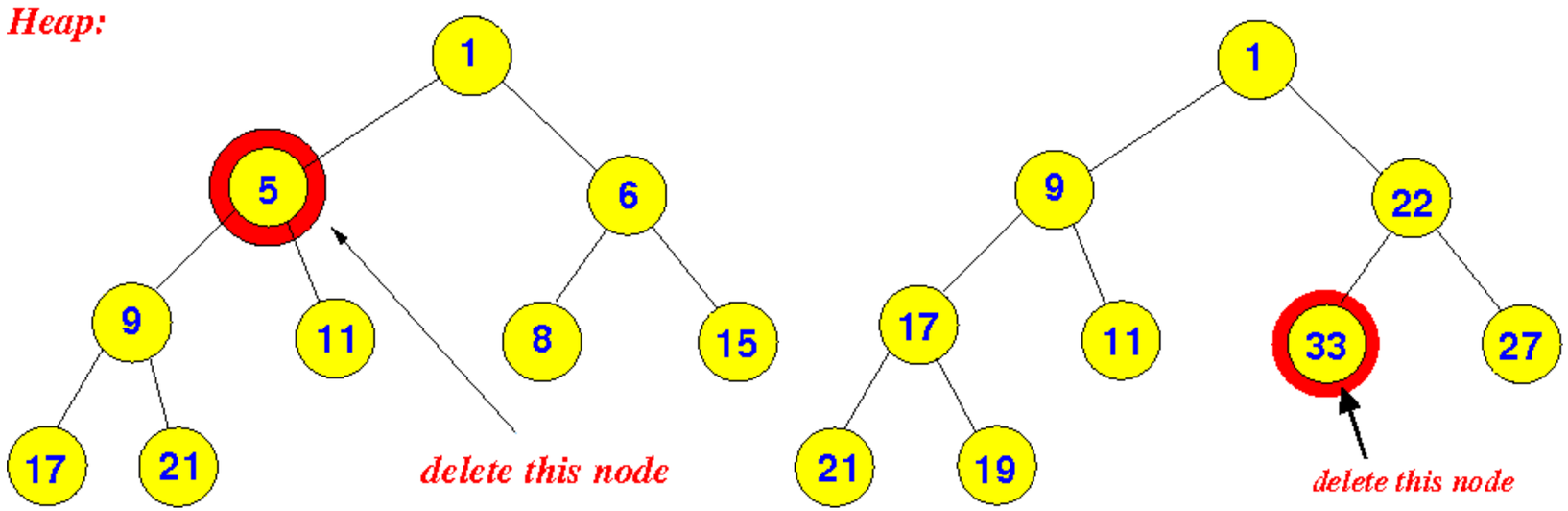
```
private void removeTop()
{
    backArray[0] = backArray[size-1];
    backArray[size-1]=null;
    size--;
    int upp=0;
    while (true)
    {
        int dwn;
        int dwn1 = upp*2+1;
        if (dwn1>size) break;
        int dwn2 = upp*2+2;
        if (dwn2>size) {  dwn=dwn1;
        } else {
            int cmp = backArray[dwn1].compareTo(backArray[dwn2]);
            if (cmp<=0)  dwn=dwn1;
            else dwn=dwn2;
        }
        if (0 > backArray[dwn].compareTo(backArray[upp]))
        {
            Pair<K,V> tmp = backArray[dwn];
            backArray[dwn] = backArray[upp];
            backArray[upp] = tmp;
            upp=dwn;            }
        else {  break;            } } }
```

Complexity Analysis

	Unordered	Ordered (using SAL)	Heap Based
offer			
peek			
poll			

General Removal

- swap with last node
- delete last node
- may need to upheap or downheap



Lab

- You are building a heap-based min priority queue with integer keys.
  - That is, the min should be at the top of the heap.
- Ignoring the values ..
- You receive the keys in this order
  - 5,6,7,3,8,1,9,4
  - Show the heap after each item is added
- Remove the min items from the heap (1)
  - show the heap after updating