Trees
Tree

- A tree is an abstract model of a hierarchical structure
- Nodes have a parent-child relation
- No loops
- One Path
Terminology
Same as for heaps

- **root**: no parent – 
  - There is only one root
- **external node/leaf**: no children –
- **internal node**: node with at least one child -
- **ancestor/descendent**
- **depth**: # of ancestors
- **Height**: max depth

• Subtree: tree consisting of a node and its descendants
Binary Tree

- An tree with every node having at most two children – left and right
Type of Binary Trees

- A binary tree is complete if every level (except possibly the last) is filled
  - A (binary) heap is a complete binary tree
  - A Complete binary tree has height $= \log_2(n)$
Binary Tree Properties

• Let $n$ denote the number of nodes and $h$ the height of a binary tree
  - $h + 1 \leq n \leq 2^{h+1} - 1$
  - $\log(n + 1) - 1 \leq h \leq n - 1$

• Height of a binary tree is usually (you hope) $O(\log n)$ of the max number of nodes
  • worst case ??
public interface TreeInterface<B> {
    int size();
    int height();
    boolean isEmpty();
    boolean contains(B element);
    void insert(B element);
    B remove(B element);
    String printNaturalOrder();
}
protected class Node<F extends Comparable<F>> {
    F payload;
    Node<F> right;
    Node<F> left;

    public Node(F e) {
        payload = e;
        right = null;
        left = null;
    }

    public String toString() {
        return payload.toString();
    }
}

This looks a lot like a doubly linked list!!
So, is a doubly linked list a tree?
public class LinkedBinaryTree<E extends Comparable<E>> implements TreeInterface<E> {

    protected Node ... 

    protected int size;
    protected Node<E> root;

Class name violates Encapsulation!
Insertion

- smaller to the left, bigger to the right

Following this pattern creates a “Binary Search Tree”
Size (number of nodes) of tree is

- size of right subtree plus
- size of left subtree plus
- 1

```java
public int size() {
    return sizeAltUtil(root);
}

private int sizeAltUtil(Node<E> treepart) {
    if (treepart == null)
        return 0;
    return sizeAltUtil(treepart.right) +
           sizeAltUtil(treepart.right) +
           1;
}
```
Height / maxDepth

Again, using a recursive helper method

```java
@Override
public int height()
{
    return maxDepthUtil(root, 1);
}

int maxDepthUtil(Node n, int depth) {
    ...
}
```
contains

- returns true if found in the tree, false otherwise
- Assumes / requires Binary search tree
Contains Algorithm

- compare with root of **current subtree**
  - root is empty – return false
  - root == element – return true
  - root < element – recurse on right child
  - root > element - recurse on left child

- Comparisons are assumed to be done using Comparable interface (ie, the compareTo method)
  - <E extends Comparable<E>>
findRec(root, key):
    if root == null:
        return false
    if root.key == key:
        return true
    if root.key > key:
        return findRec(root.left, key)
    else
        return findRec(root.right, key)
Contains Code

• Write using a recursive helper method

```java
public boolean contains(E element) {
    if (root==null) return false;
    return containsUtil(root, element)!="null";
}

private Node containsUtil(Node treepart, E toBeFound) {
    ...
}
```
Unordered Contains

- Suppose that you did not know relation among children (you do NOT have a binary search tree)
  - So thing being looked for could be either left or right
  - How would you change containsUtil function
    - Would a tree be a useful structure in this case?
insert

- void insert(E element);
- new node is always inserted as a leaf
- inserts to
  - left subtree if element is smaller than subtree root
  - right subtree if larger
  - Pre-case: if root=null then root=new Node

```java
public void insert(E element) {
    if (root==null) {
        root=new Node<>(element);
        size = 1;
    } else
        insertUtil(root, element);
}
```
Groups

- Draw binary search trees for data received from left to right
  - 4, 5, 6, 49, 43, 31, 19, 10, 11, 8, 17
  - 17, 31, 8, 19, 43, 11, 5, 49, 10, 6, 4

- Write insertUtil

```java
private void insertUtil(Node treepart, E toBeAdded) {
  ... }
```
Traversals / Printing
public void printPostOrder() {
    iPrintPostOrder(root, 0);
    System.out.println();
}

private void iPrintPostOrder(Node treePart, int depth) {
    if (treePart == null) return;
    iPrintPostOrder(treePart.left, depth+1);
    iPrintPostOrder(treePart.right, depth+1);
    System.out.print("["+treePart.payload+","+depth+"]");
}
Remove

• boolean remove(E element);

• returns true if element existed and was removed and false otherwise

• Cases
  □ element not in tree
  □ element is a leaf
  □ element has one child
  □ element has two children
Leaf

- Just delete
One child

- Replace with child – skip over like in linked list
Two Children

• Replace with in-order predecessor or in-order successor

• in-order predecessor
  □ rightmost child in left subtree
  □ max-value child in left subtree

• in-order successor
  □ leftmost child in right subtree
  □ min-value child in right subtree
Replace with Predecessor
Replace with Successor
Practice

• Given the data:
  • 6, 19, 10, 5, 43, 31, 11, 8, 4, 17, 49, 36

• Draw the binary tree
• Write the preorder traversal of your tree
• Write the postorder traversal of your tree
• What the height of the tree?
• If the data were re-arranged, what is the shortest possible tree?