Review

- Transformations
  - Scale
  - Translate
  - Rotate
- Combining Transformations
  - Transformations are cumulative
  - Rotating about the center of an object
- Animating with transformations

Factorial

- The factorial of a positive integer N is computed as the product of N with all positive integers less than or equal to N.

$4! = 4 \times 3 \times 2 \times 1 = 24$

$30! = 30 \times 29 \times \ldots \times 2 \times 1 =$
$265252859812191058636308480000000$

Factorial - Iterative Implementation

```
1.   void setup() {
2.     int A = 10;
3.     int B = factorial(5);
4.     println( B );
5.   }

6.   int factorial(int N) {
7.     int F = 1;
8.
9.     for( int i=N; i>=1; i--) {
10.      F = F * i;
11.    }
12.
13.    return F;
14.  }
```

Trace it.

$5! = 5 \times 4 \times 3 \times 2 \times 1$
$4! = 4 \times 3 \times 2 \times 1$

$5! = 5 \times 4!$

$N! = N \times (N-1)!$

Factorial can be defined in terms of itself

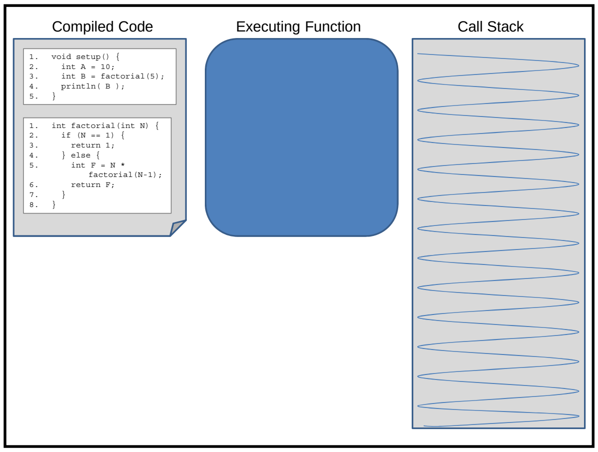
Factorial – Recursive Implementation

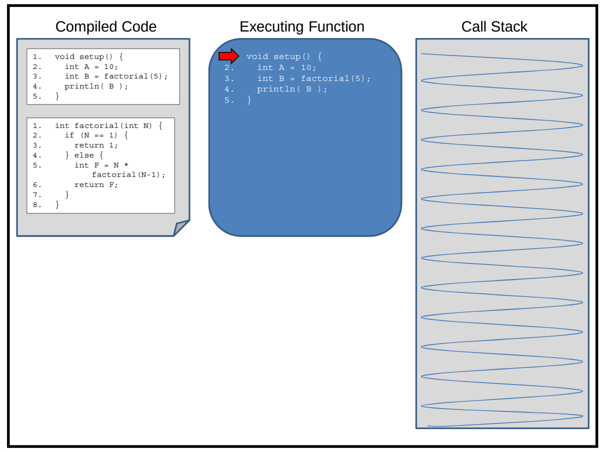
```
1.   void setup() {
2.     int A = 10;
3.     int B = factorial(5);
4.     println( B );
5.   }

6.   int factorial(int N) {
7.     if (N == 1) {
8.       return 1;
9.     } else {
10.      int F = N * factorial(N-1);
11.      return F;
12.    }
13.  }
```

Trace it.

Last In First Out (LIFO) Stack of Plates


Compiled Code
Executing Function
Call Stack
1. void setup() {
2. int A = 10;
3. int B = factorial(5);
4. println(B);
5. }
1. int factorial(int N) {
2. if (N == 1) {
3. return 1;
4. } else {
5. int F = N * factorial(N-1);
6. return F;
7. }
8. }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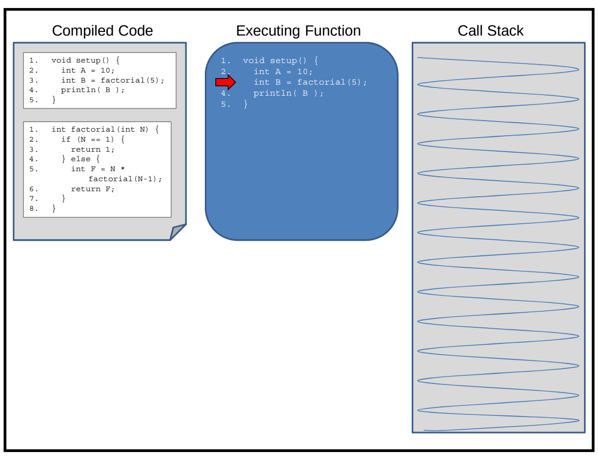
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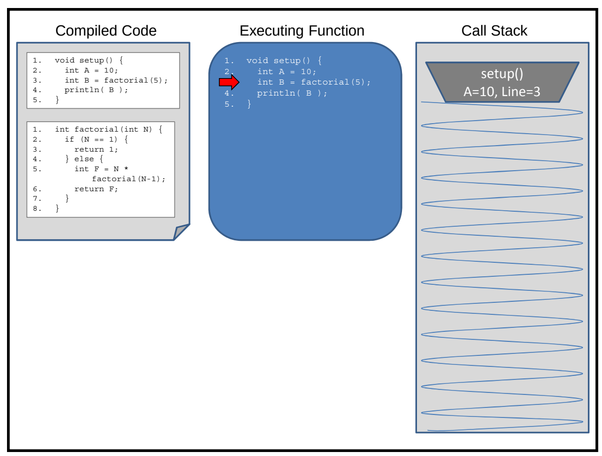
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Executing Function
Call Stack
setup()
A=10, Line=3
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2. int A = 10;
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5. }
1. int factorial(int N) {
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3. return 1;
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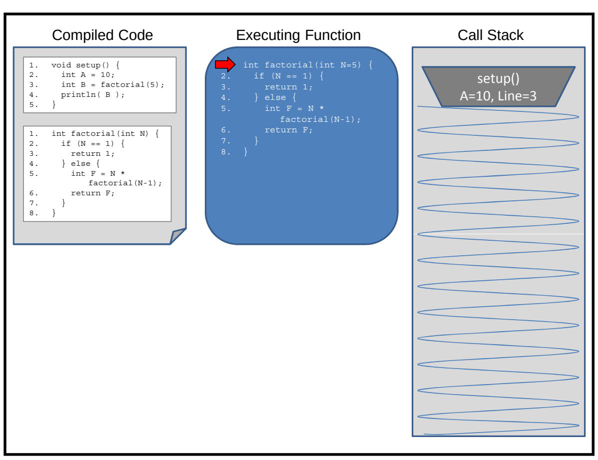
5. int F = N * factorial(N-1);
6. return F;
7. }
8. }


Compiled Code
Executing Function
Call Stack
setup()
A=10, Line=3
1. int factorial(int N=5) {
2. if (N == 1) {
3. return 1;
4. } else {
5. int F = N * factorial(N-1);
6. return F;
7. }
8. }
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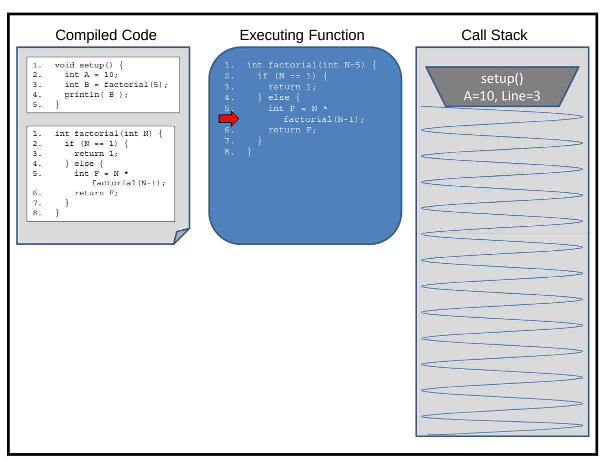
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Call Stack
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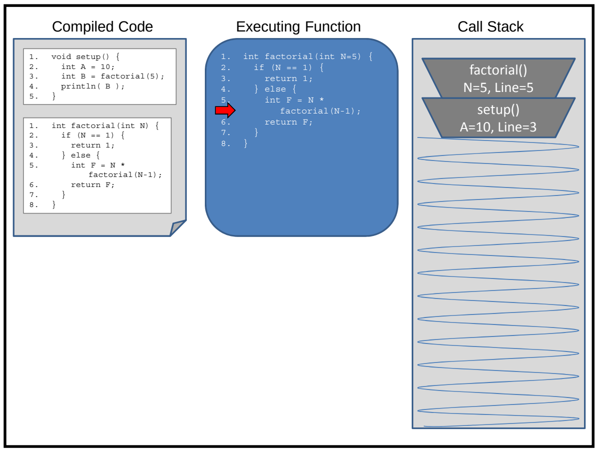
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3. int B = factorial(5);
4. println(B);
5. }
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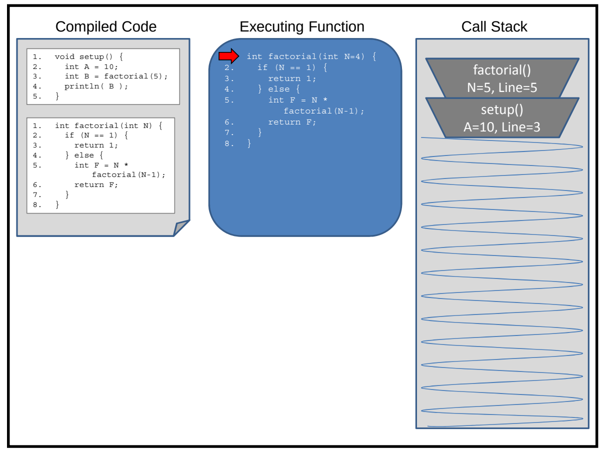
2. if (N == 1) {
3. return 1;
4. } else {
5. int F = N *
factorial(N-1);
6. return F;
7. }
8. }
Executing Function
1. int factorial(int N=5) {
2. if (N == 1) {
3. return 1;
4. } else {
5. int F = N *
factorial(N-1);
6. return F;
7. }
8. }
Call Stack
factorial()
N=5, Line=5
setup()
A=10, Line=3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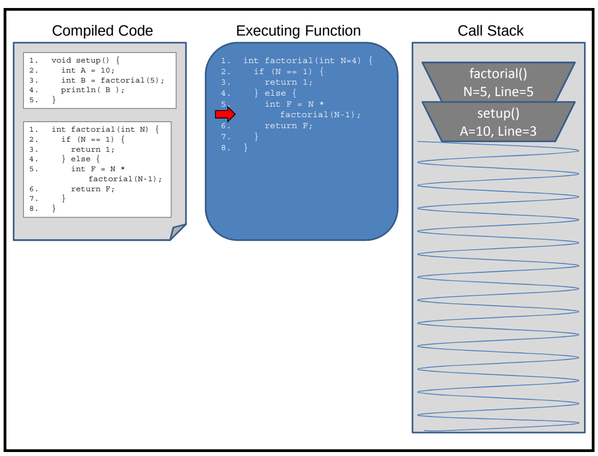
2. int A = 10;
3. int B = factorial(5);
4. println(B);
5. }
1. int factorial(int N) {
2. if (N == 1) {
3. return 1;
4. } else {
5. int F = N *
factorial(N-1);
6. return F;
7. }
8. }
Executing Function
1. int factorial(int N=4) {
2. if (N == 1) {
3. return 1;
4. } else {
5. int F = N *
factorial(N-1);
6. return F;
7. }
8. }
Call Stack
factorial()
N=5, Line=5
setup()
A=10, Line=3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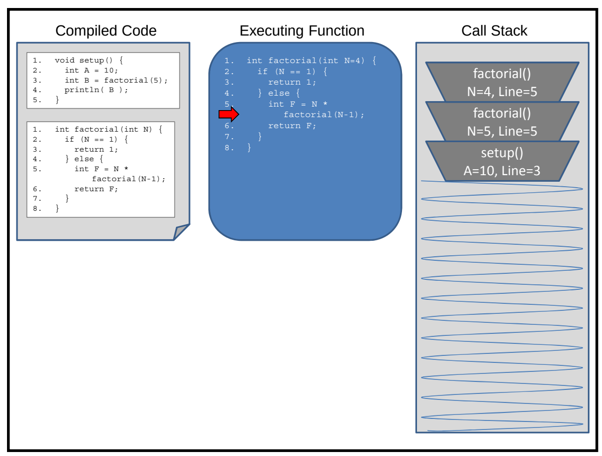
2. int A = 10;
3. int B = factorial(5);
4. println(B);
5. }
1. int factorial(int N) {
2. if (N == 1) {
3. return 1;
4. } else {
5. int F = N *
factorial(N-1);
6. return F;
7. }
8. }
Executing Function
1. int factorial(int N=4) {
2. if (N == 1) {
3. return 1;
4. } else {
5. int F = N *
factorial(N-1);
6. return F;
7. }
8. }
Call Stack
factorial()
N=5, Line=5
setup()
A=10, Line=3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Call Stack
factorial()
N=4, Line=5
factorial()
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setup()
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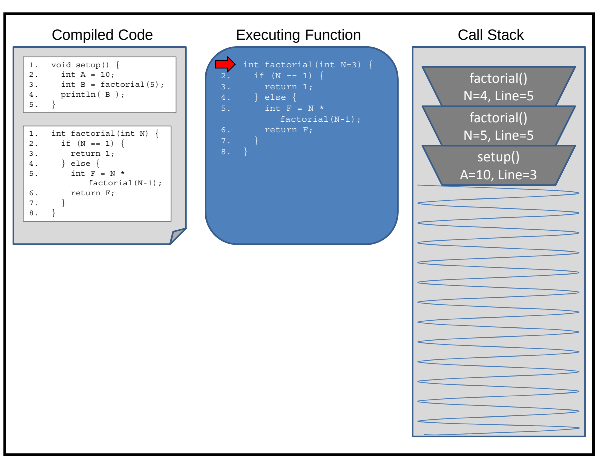
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4. } else {
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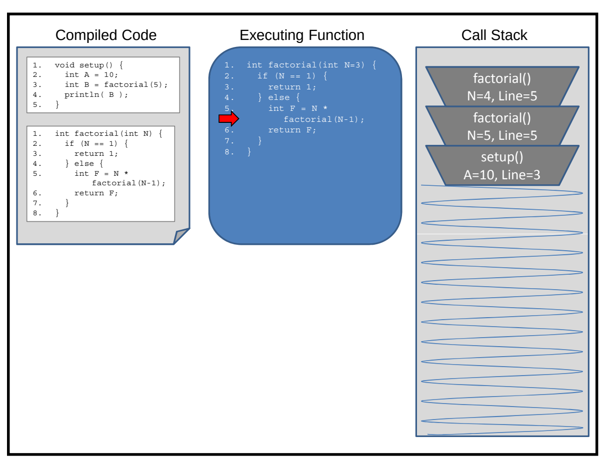
factorial(N-1);
6. return F;
7. }
8. }
Executing Function
1. int factorial(int N=3) {
2. if (N == 1) {
3. return 1;
4. } else {
5. int F = N *
factorial(N-1);
6. return F;
7. }
8. }
Call Stack
factorial()
N=4, Line=5
factorial()
N=5, Line=5
setup()
A=10, Line=3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4. println(B);
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3. return 1;
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5. int F = N *
factorial(N-1);
6. return F;
7. }
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1. int factorial(int N=3) {
2. if (N == 1) {
3. return 1;
4. } else {
5. int F = N *
factorial(N-1);
6. return F;
7. }
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Call Stack
factorial()
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factorial()
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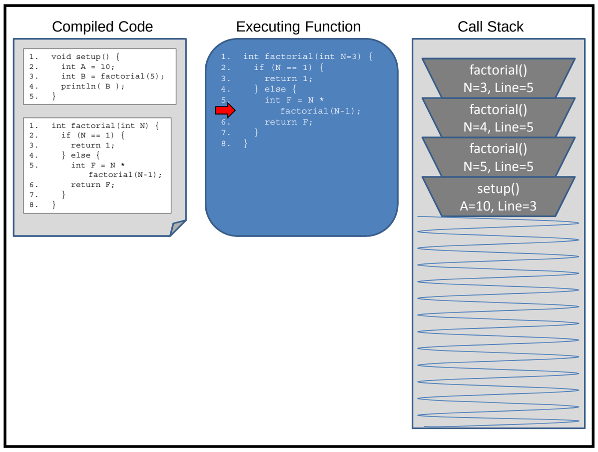
setup()
A=10, Line=3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4. } else {
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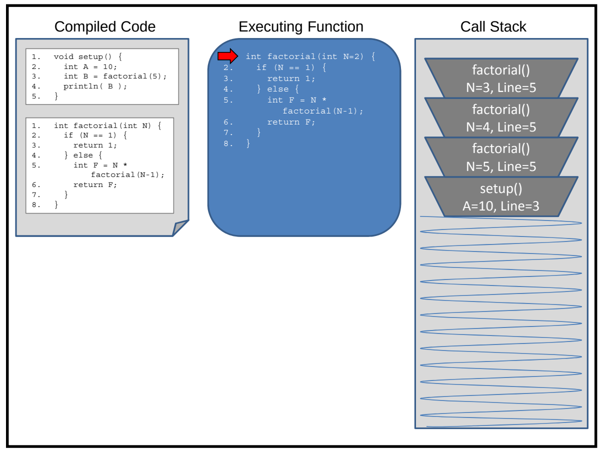
6. return F;
7. }
8. }
Call Stack
factorial() N=3, Line=5
factorial() N=4, Line=5
factorial() N=5, Line=5
setup() A=10, Line=3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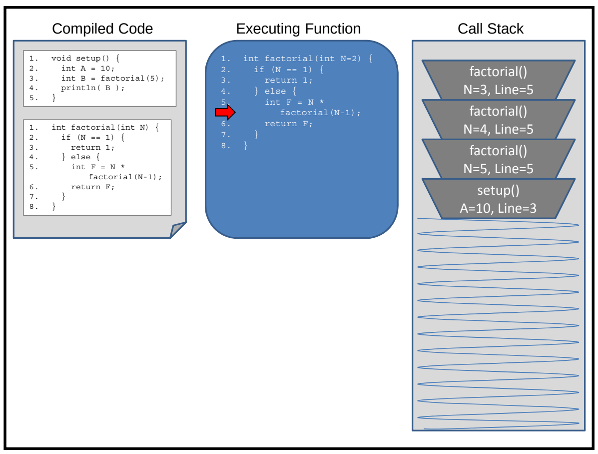
3. int B = factorial(5);
4. println(B);
5. }
1. int factorial(int N) {
2. if (N == 1) {
3. return 1;
4. } else {
5. int F = N * factorial(N-1);
6. return F;
7. }
8. }
Executing Function
1. int factorial(int N=2) {
2. if (N == 1) {
3. return 1;
4. } else {
5. int F = N * factorial(N-1);
6. return F;
7. }
8. }
Call Stack
factorial() N=3, Line=5
factorial() N=4, Line=5
factorial() N=5, Line=5
setup() A=10, Line=3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4. println(B);
5. }
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3. return 1;
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5. int F = N * factorial(N-1);
6. return F;
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1. int factorial(int N=2) {
2. if (N == 1) {
3. return 1;
4. } else {
5. int F = N * factorial(N-1);
6. return F;
7. }
8. }
Call Stack
factorial() N=3, Line=5
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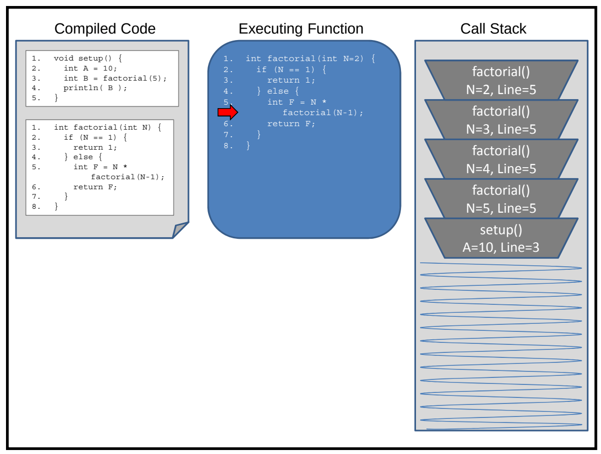
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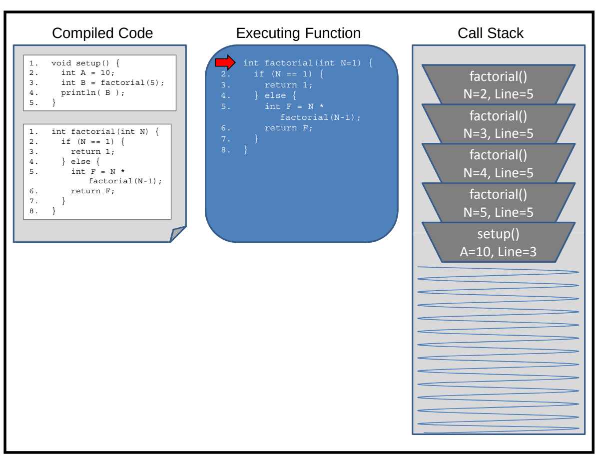
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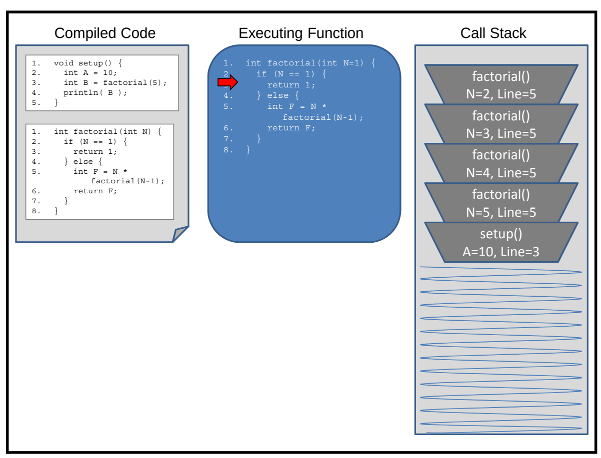
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2. if (N == 1) {
3. return 1;
4. } else {
5. int F = N * factorial(N-1);
6. return F;
7. }
8. }
Executing Function
1. int factorial(int N=1) {
2. if (N == 1) {
3. return 1;
4. } else {
5. int F = N * factorial(N-1);
6. return F;
7. }
8. }
Call Stack
factorial() N=2, Line=5
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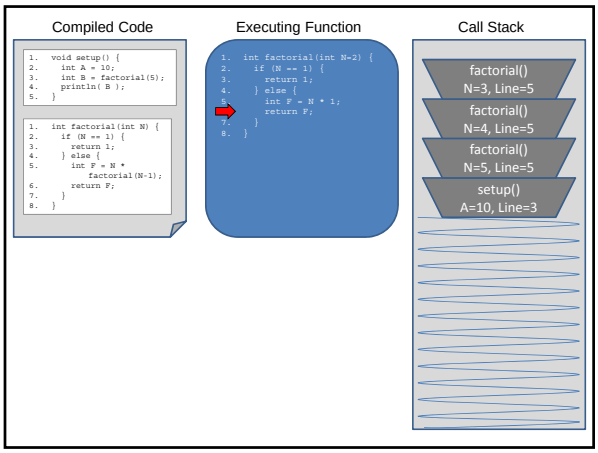
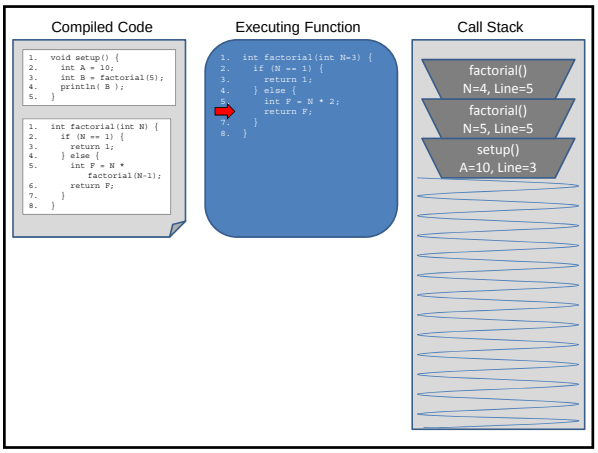
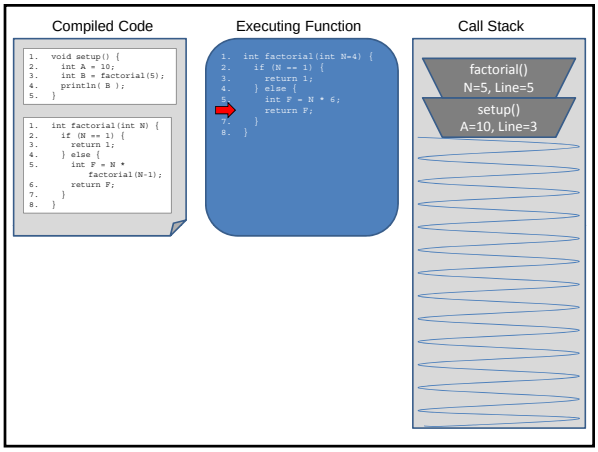
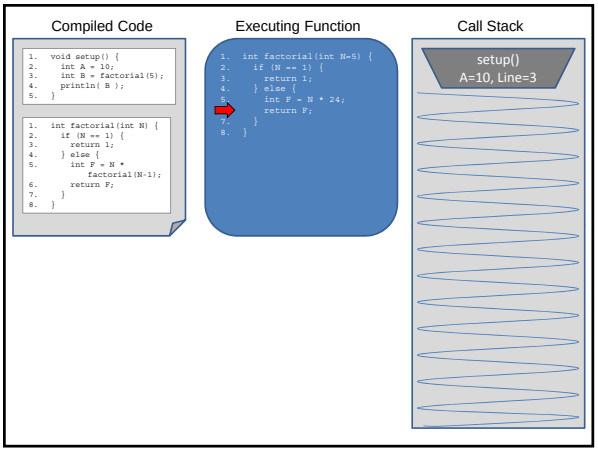
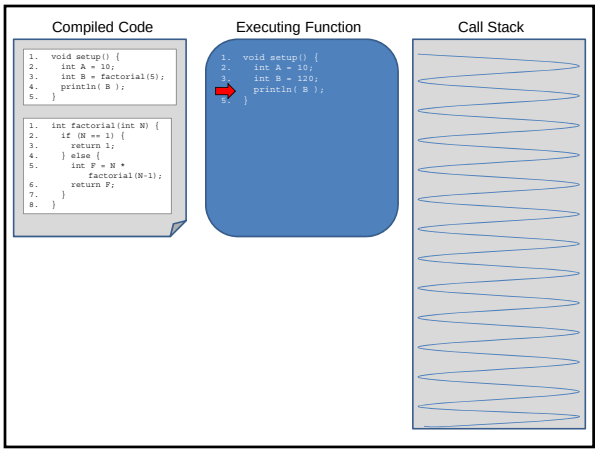






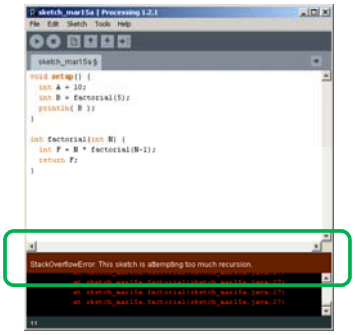




The Call Stack keeps track of …

1. all functions that are suspended, in order
2. the point in the function where execution should resume after the invoked subordinate function returns
3. a snapshot of all variables and values within the scope of the suspended function so these can be restored upon continuing execution

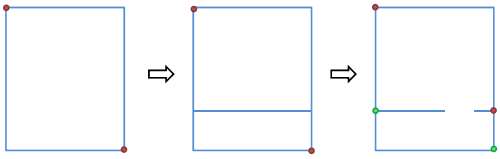
What happens if there is no stopping condition, or "base case"?



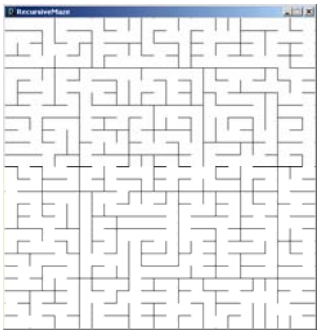

```
// Fibonacci sequence
// 0, 1, 1, 2, 3, 5, 8, 13, 21, 34, 55, 89, 144, ...

void setup() {}
void draw() {}

void mousePressed() {
  int f = fibonacci(12);
  println(f);
}

// Compute and return the nth Fibonacci number
int fibonacci(int n) {
  if (n == 0) {
    return 0;
  } else if (n == 1) {
    return 1;
  } else {
    int f = fibonacci(n-1) + fibonacci(n-2);
    return f;
  }
}
```

Creating a maze, recursively

1. Start with a rectangular region defined by its upper left and lower right corners
2. Divide the region at a random location through its more narrow dimension
3. Add an opening at a random location
4. Repeat on two rectangular subregions



Inspired by http://weblog.jamisbuck.org/2011/1/12/maze-generation-recursive-division-algorithm



```
// RecursiveMaze

int N = 25;     // Grid dimension
int gsize = 20; // Grid size

int V = 1;      // Vertical constant
int H = 2;      // Horizontal constant

void setup() {
  // Setup sketch
  size(N*gsize+1, N*gsize+1);
  noLoop();
  background(255);
  stroke(0);

  // Kick off the recursive divide
  // on entire sketch window
  divide(0,0,N,N);
}

// Determine the direction for dividing
// Stop when too small.
int divDir(int r1, int c1, int r2, int c2) {
  int dr = r2 - r1;         // Deltas
  int dc = c2 - c1;
  if (dr <= 1 || dc <= 1)   // Too small
    return 0;               // No division
  else if (dr < dc)         // Flat and wide
    return V;               // Vertical division
  else                      // Tall and narrow
    return H;               // Horizontal div
}

// Return a random integer in the range
int randomInt(int min, int max) {
  return round(random(min-0.5,max+0.5));
}

// Draw a line on a grid segment
void gridLine(int r1, int c1, int r2, int c2) {
  line(r1*gsize, c1*gsize, r2*gsize, c2*gsize);
}
```

```
// Divide the region given upper left and
// lower right grid corner points

void divide(int r1, int c1, int r2, int c2)
{
  int cr, rr;

  // Get divide direction (V, H or 0)
  int dir = divDir(r1, c1, r2, c2);

  // Divide in vertical direction
  if (dir == V) {

    // Wall and opening locations
    cr = randomInt(c1+1, c2-1);
    rr = randomInt(r1, r2-1);

    // Draw wall
    gridLine(cr,r1,cr,rr);
    gridLine(cr,rr+1,cr,r2);

    // Recursively divide two subregions
    divide(r1,c1,r2,cr);
    divide(r1,cr,r2,c2);

  // Divide in horizontal direction
  } else if (dir == H) {

    // Wall and opening locations
    cr = randomInt(c1, c2-1);
    rr = randomInt(r1+1, r2-1);

    // Draw wall
    gridLine(c1,rr,cr,rr);
    gridLine(cr+1,rr,c2,rr);

    // Recursively divide two subregions
    divide(r1,c1,rr,c2);
    divide(rr,c1,r2,c2);

  // No division. We're done.
  } else {
    return;
  }
}
```