Recursion

What is recursion?

- Recursion is the ability for a function to call itself.
- Why would we want to call our own function?

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- Why would we want to call our own function?
- Here's an example of a recursive function:
- ```
  void recursiveCount(int start) {
    println(start);
    recursiveCount(start+1);
  }
  ```
  ```

What is recursion?

- Recursion is the ability for a function to call itself.
- Why would we want to call our own function?
- Here's an example of a recursive function:
- ```
  void recursiveCount(int start) {
    println(start);
    recursiveCount(start+1);
  }
  ```
- What does it do?
  ```

Iteration vs. Recursion

- Here's a simlar iterative function:
- ```
  void iterativeCount(int start) {
    while(true) {
      println(start++);
    }
  }
  ```
  ```

Iteration vs. Recursion

- Here's a simlar iterative function:
- ```
  void iterativeCount(int start) {
    while(true) {
      println(start++);
    }
  }
  ```
- How does the above compare to:
- ```
  void recursiveCount(int start) {
     println(start);
     recursiveCount(start+1);
  }
  ```

Okay, so what is it good for?

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  - simple, that's just 5 x 4 x 3 x 2 x 1
- What about 30! ?
  - still simple, that's just 30 x 29 x … x 2 x 1

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- What about N! ?

Okay, so what is it good for?

- Solving problems by breaking them into smaller pieces.
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  - simple, that's just 5 x 4 x 3 x 2 x 1
- What about 30! ?
  - still simple, that's just 30 x 29 x … x 2 x 1
- What about N! ?
  - Let's use a loop (iteration).

Okay, so what is it good for?

- Solving problems by breaking them into smaller pieces.
- For example how do you get 5! (five factorial)?
  - simple, that's just 5 x 4 x 3 x 2 x 1
- What about 30! ?
  - still simple, that's just 30 x 29 x … x 2 x 1
- What about N! ?
  - Let's use a loop (iteration).
  - We could also use recursion.

Recursion – Key idea

- Keep delegating until the problem is very simple.
- In other words
  - do a simpler task each time you call yourself
  - until the task would cause no change (the base case).

N! recursive definition

- Recursive case:
  - N! is N * (N-1)!
- Base case:
  - 0! is 1
- Note: N-1 is a simpler/smaller version of the problem for factorial to act upon.

N! recursive definition

- Recursive case (let's break this down):
  - M = N-1;// make the problem smaller
  - smallerFact = M! // call factorial on the smaller problem
  - return N * smallerFact; // return result of simple multiplication.
- Base case (if N == 0):
  - return 1;
- Note: N-1 is a simpler/smaller version of the problem for factorial to act upon.

+

N! recursive definition

- Recursive case:
  - N! is N * (N-1)!
- Base case:
  - 0! is 1

- So, what does this mean for 5!?
- How could we write a function to capture these 2 cases?

Factorial – Recursive Implementation

```
void setup() {
  int A = 10;
  int B = factorial(5);
  println( B );
}

int factorial(int N) {
  if (N == 1) {
    return 1;
  } else {
    int F = N * factorial(N-1);
    return F;
  }
}
```

Trace it.

+

Last In First Out (LIFO) Stack of Plates

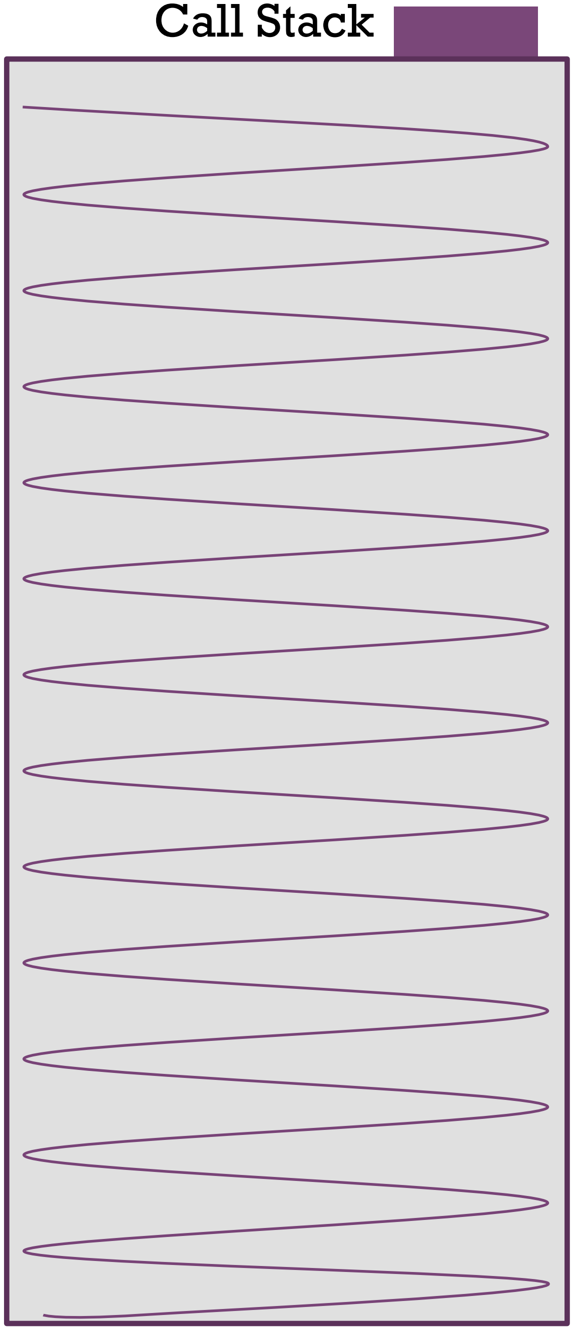
Compiled Code

```
void setup() {
  int A = 10;
  int B = factorial(5);
  println( B );
}
```

```
int factorial(int N) {
  if (N == 1) {
    return 1;
  } else {
    int F = N * factorial(N-1);
    return F;
  }
}
```

Executing Function

Call Stack



Compiled Code

```
void setup() {
  int A = 10;
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  println( B );
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```
int factorial(int N) {
  if (N == 1) {
    return 1;
  } else {
    int F = N *
 factorial(N-1);
    return F;
  }
}
```

Executing Function

```
void setup() {
  int A = 10;
  int B = factorial(5);
  println( B );
}
```

Call Stack

Compiled Code

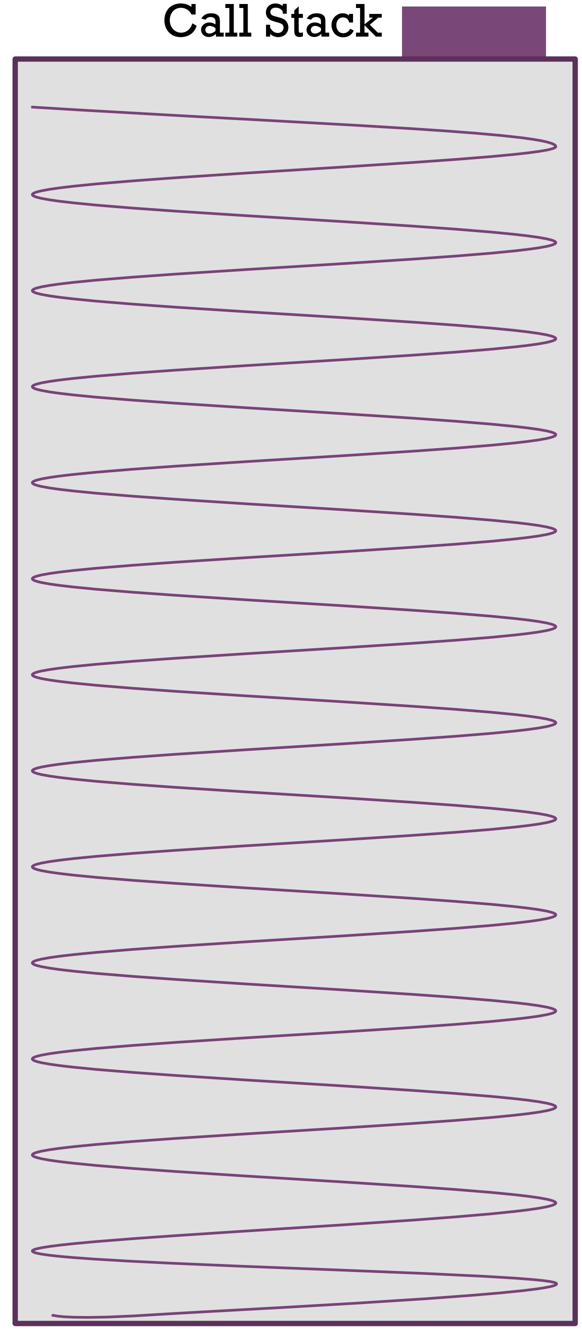
```
void setup() {
  int A = 10;
  int B = factorial(5);
  println( B );
}
```

```
int factorial(int N) {
  if (N == 1) {
    return 1;
  } else {
    int F = N *
  factorial(N-1);
    return F;
  }
}
```

Executing Function

```
void setup() {
  int A = 10;
  int B = factorial(5);
  println( B );
}
```

Call Stack



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2.     int A = 10;
3.     int B = factorial(5);
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5.   }
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3.       return 1;
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5.       int F = N *
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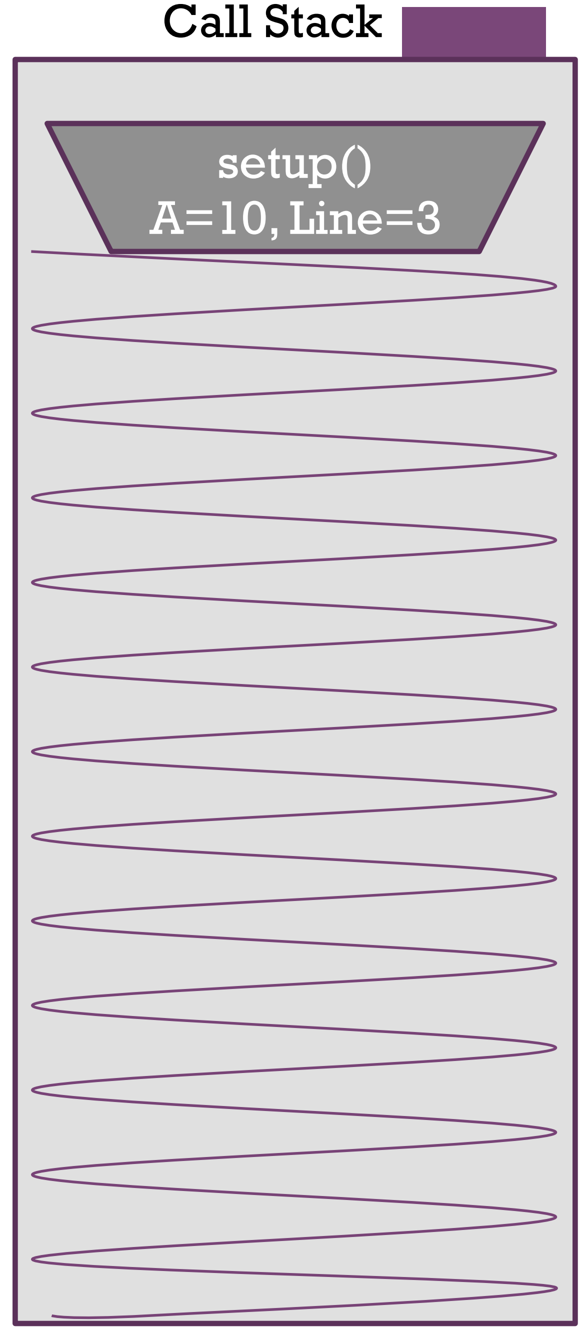
6.       return F;
7.     }
8.   }
```

Executing Function

```
1.   void setup() {
2.     int A = 10;
3.     int B = factorial(5);
4.     println( B );
5.   }
```

Call Stack



Compiled Code

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2.     int A = 10;
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Executing Function

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     int factorial(int N=5) {
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       factorial(N-1);
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```
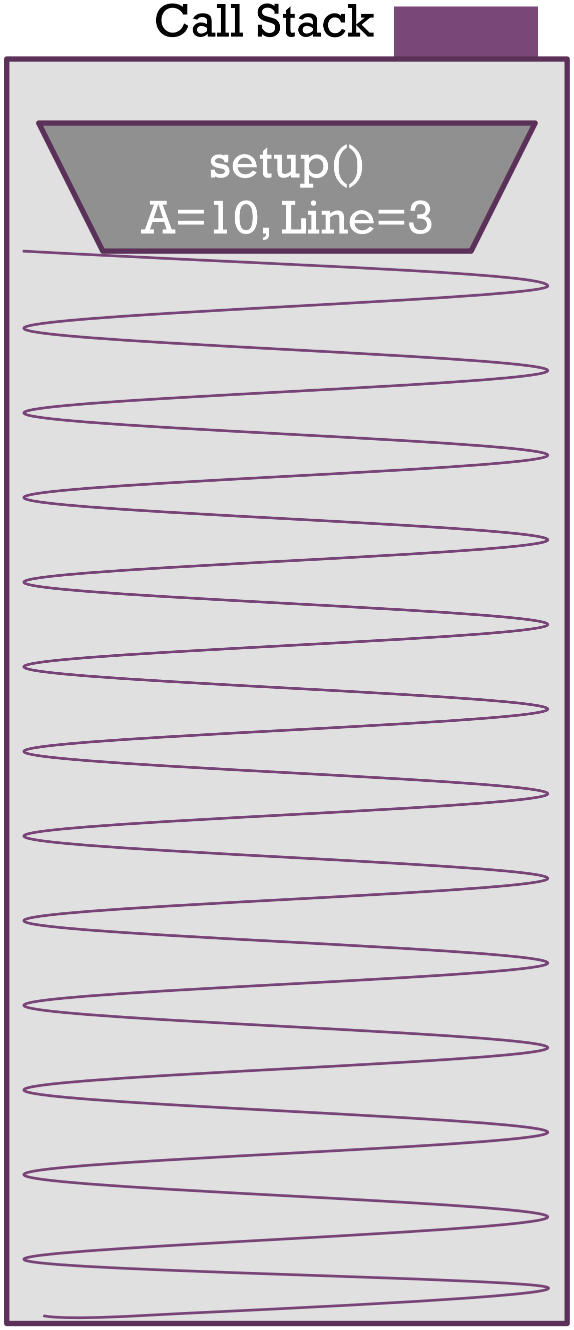

Call Stack



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2.     int A = 10;
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1.   int factorial(int N) {
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Executing Function

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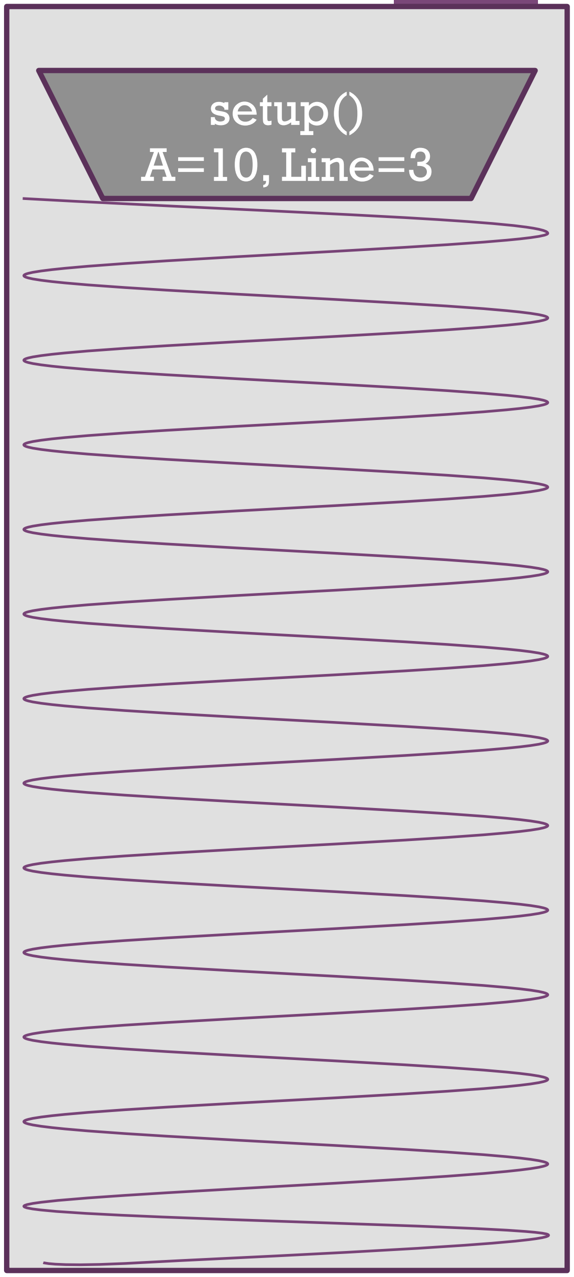
Call Stack



Compiled Code

```
1.   void setup() {
2.     int A = 10;
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Executing Function

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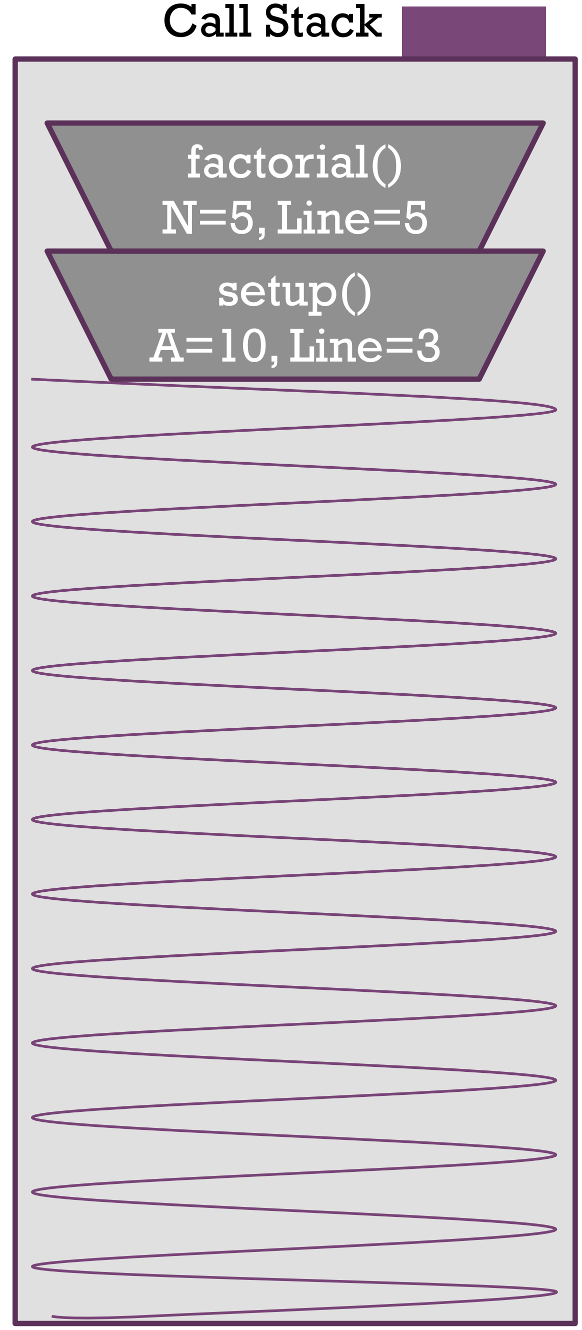
Call Stack



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3.       return 1;
4.     } else {
5.       int F = N *
        factorial(N-1);
6.       return F;
7.     }
8.   }
```

Executing Function

```
     int factorial(int N=4) {
2.     if (N == 1) {
3.       return 1;
4.     } else {
5.       int F = N *
       factorial(N-1);
6.       return F;
7.     }
8.   }
```

Call Stack

factorial()
N=5, Line=5

setup()
A=10, Line=3

Compiled Code

```
1.   void setup() {
2.     int A = 10;
3.     int B = factorial(5);
4.     println( B );
5.   }
```

```
1.   int factorial(int N) {
2.     if (N == 1) {
3.       return 1;
4.     } else {
5.       int F = N *
        factorial(N-1);
6.       return F;
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8.   }
```

Executing Function

```
1.   int factorial(int N=4) {
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3.       return 1;
4.     } else {
5.       int F = N *
       factorial(N-1);
6.       return F;
7.     }
8.   }
```

Call Stack

factorial()
N=5, Line=5

setup()
A=10, Line=3

Compiled Code

```
1.   void setup() {
2.     int A = 10;
3.     int B = factorial(5);
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Executing Function

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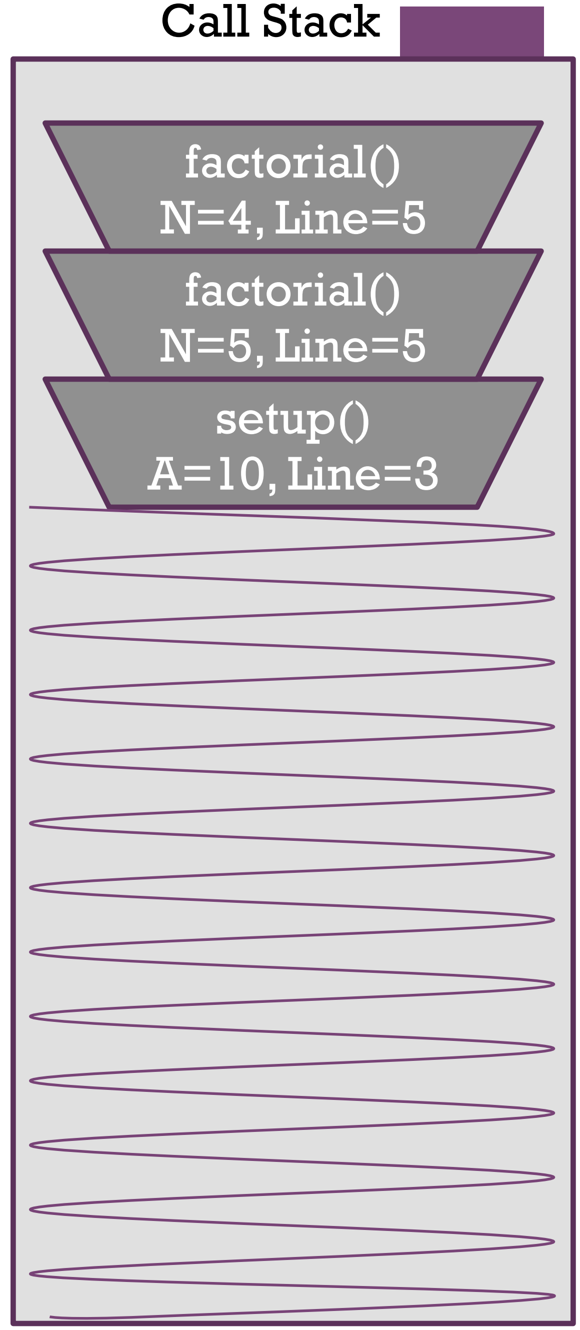
Call Stack



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Executing Function

```
     int factorial(int N=3) {
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3.       return 1;
4.     } else {
5.       int F = N *
       factorial(N-1);
6.       return F;
7.     }
8.   }
```

Call Stack

factorial()
N=4, Line=5

factorial()
N=5, Line=5

setup()
A=10, Line=3

Compiled Code

```
1.   void setup() {
2.     int A = 10;
3.     int B = factorial(5);
4.     println( B );
5.   }
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1.   int factorial(int N) {
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Call Stack

factorial()
N=4, Line=5

factorial()
N=5, Line=5

setup()
A=10, Line=3

Compiled Code

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Call Stack

factorial()
N=3, Line=5

factorial()
N=4, Line=5

factorial()
N=5, Line=5

setup()
A=10, Line=3

Compiled Code

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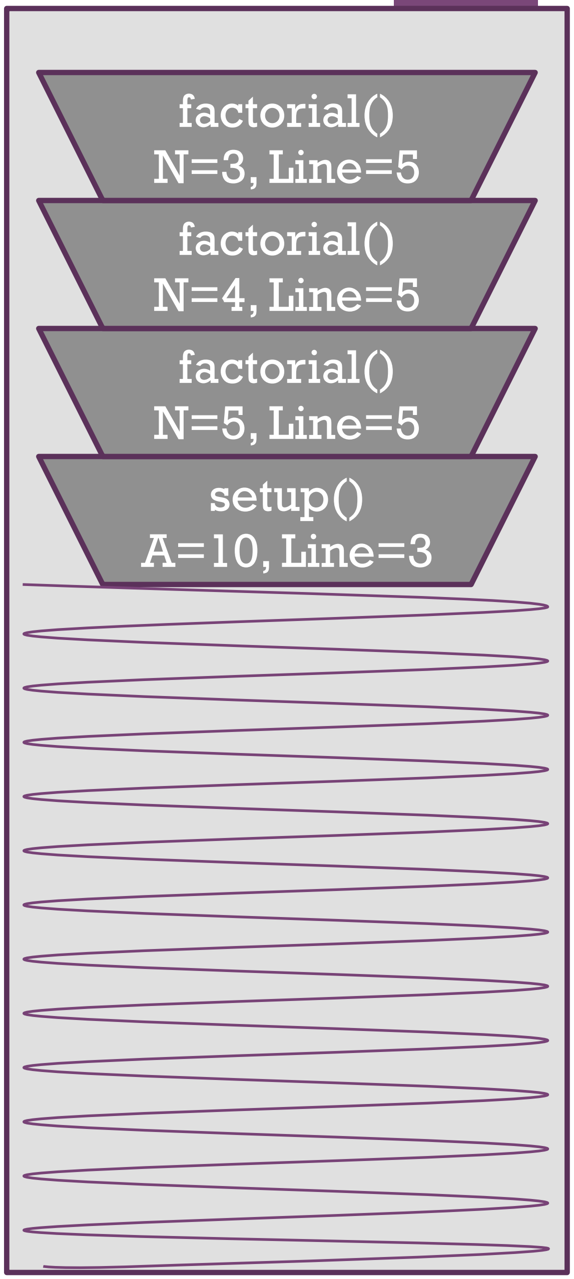
```

Executing Function

```
     int factorial(int N=2) {
2.     if (N == 1) {
3.       return 1;
4.     } else {
5.       int F = N *
       factorial(N-1);
6.       return F;
7.     }
8.   }
```

Call Stack



Compiled Code

```
1.   void setup() {
2.     int A = 10;
3.     int B = factorial(5);
4.     println( B );
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```
1.   int factorial(int N) {
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Executing Function

```
1.   int factorial(int N=2) {
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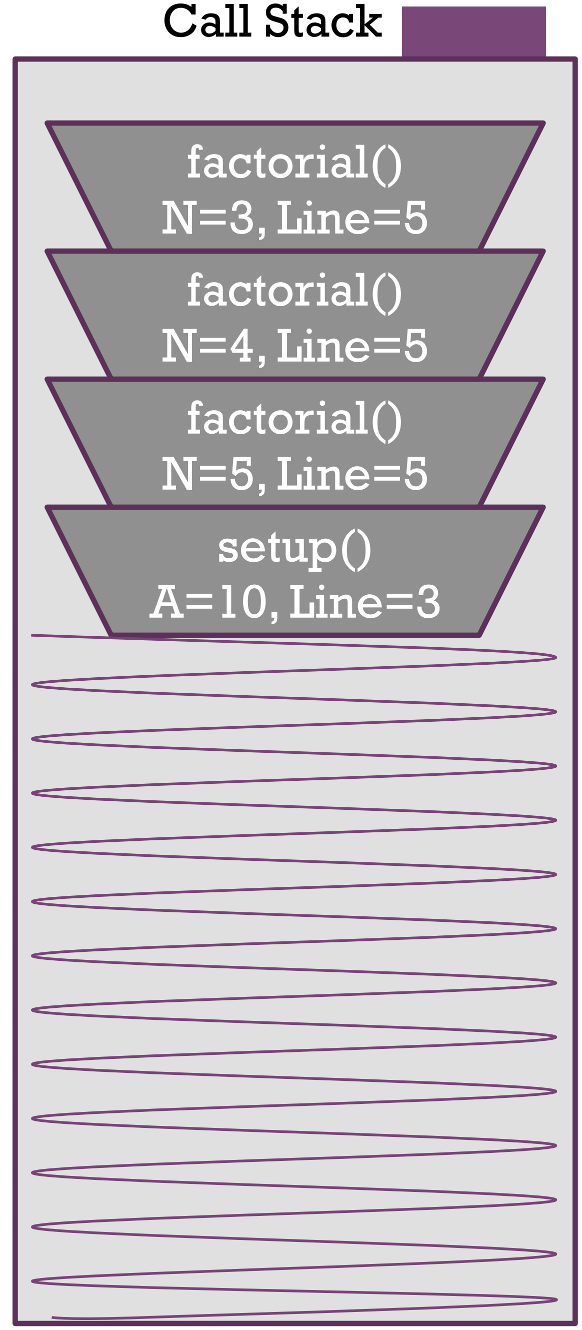
5.       int F = N *
       factorial(N-1);
6.       return F;
7.     }
8.   }
```

Call Stack



Compiled Code

```
void setup() {
  int A = 10;
  int B = factorial(5);
  println( B );
}
```

```
int factorial(int N) {
  if (N == 1) {
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  } else {
    int F = N * factorial(N-1);
    return F;
  }
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Executing Function

```
int factorial(int N=2) {
  if (N == 1) {
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    return F;
  }
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```
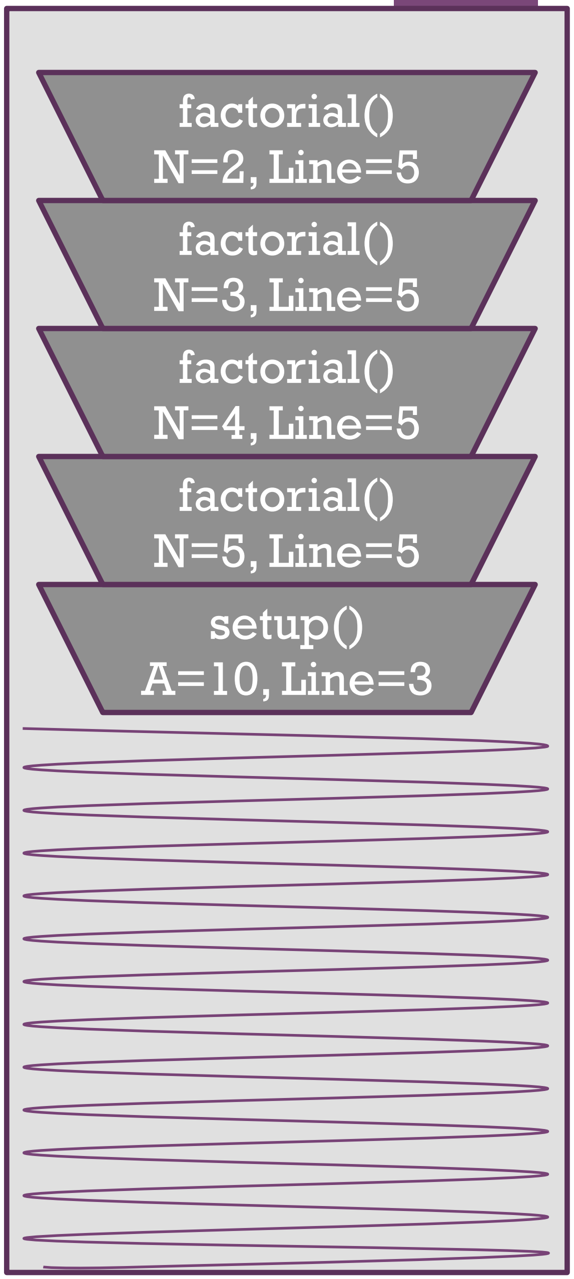

Call Stack



Compiled Code

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1.   void setup() {
2.     int A = 10;
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4.     println( B );
5.   }
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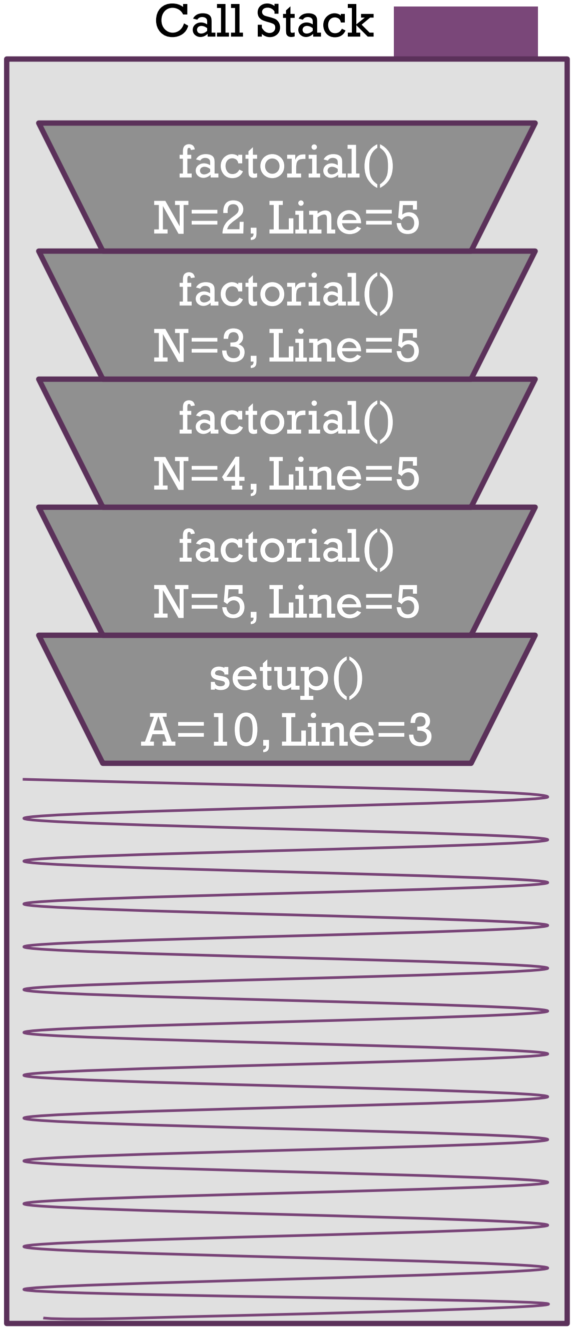
```
1.   int factorial(int N) {
2.     if (N == 1) {
3.       return 1;
4.     } else {
5.       int F = N *
       factorial(N-1);
6.       return F;
7.     }
8.   }
```

Executing Function

```
     int factorial(int N=1) {
2.     if (N == 1) {
3.       return 1;
4.     } else {
5.       int F = N *
       factorial(N-1);
6.       return F;
7.     }
8.   }
```

Call Stack



Compiled Code

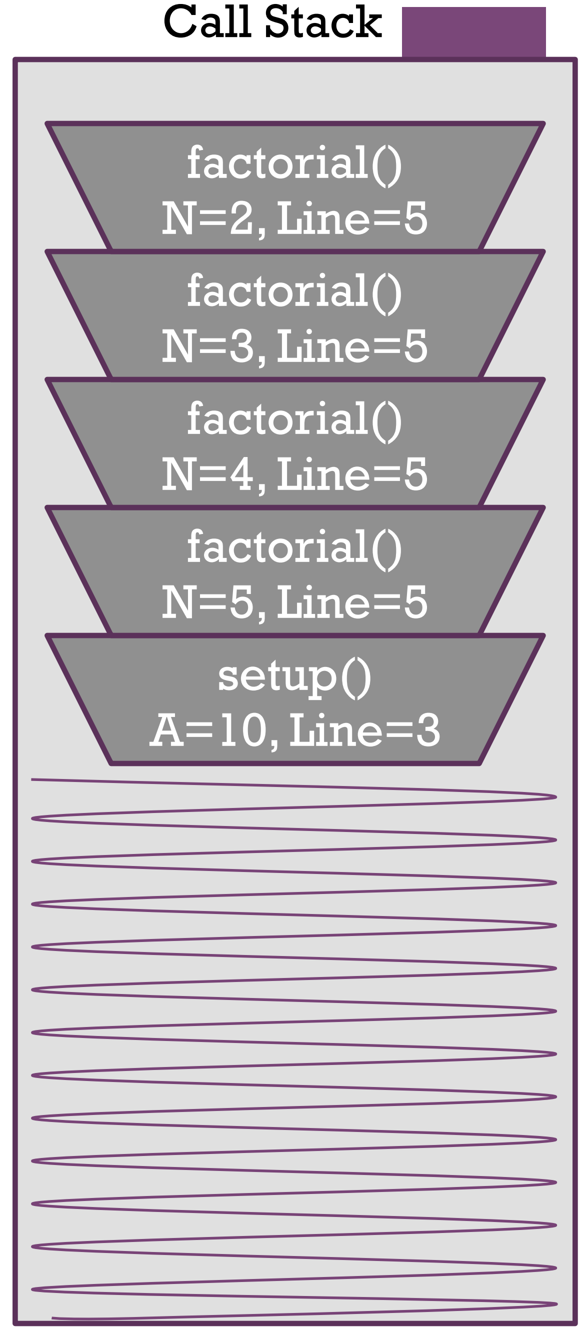
```
1.   void setup() {
2.     int A = 10;
3.     int B = factorial(5);
4.     println( B );
5.   }
```

```
1.   int factorial(int N) {
2.     if (N == 1) {
3.       return 1;
4.     } else {
5.       int F = N *
       factorial(N-1);
6.       return F;
7.     }
8.   }
```

Executing Function

```
1.   int factorial(int N=1) {
2.     if (N == 1) {
3.       return 1;
4.     } else {
5.       int F = N *
       factorial(N-1);
6.       return F;
7.     }
8.   }
```

Call Stack



Compiled Code

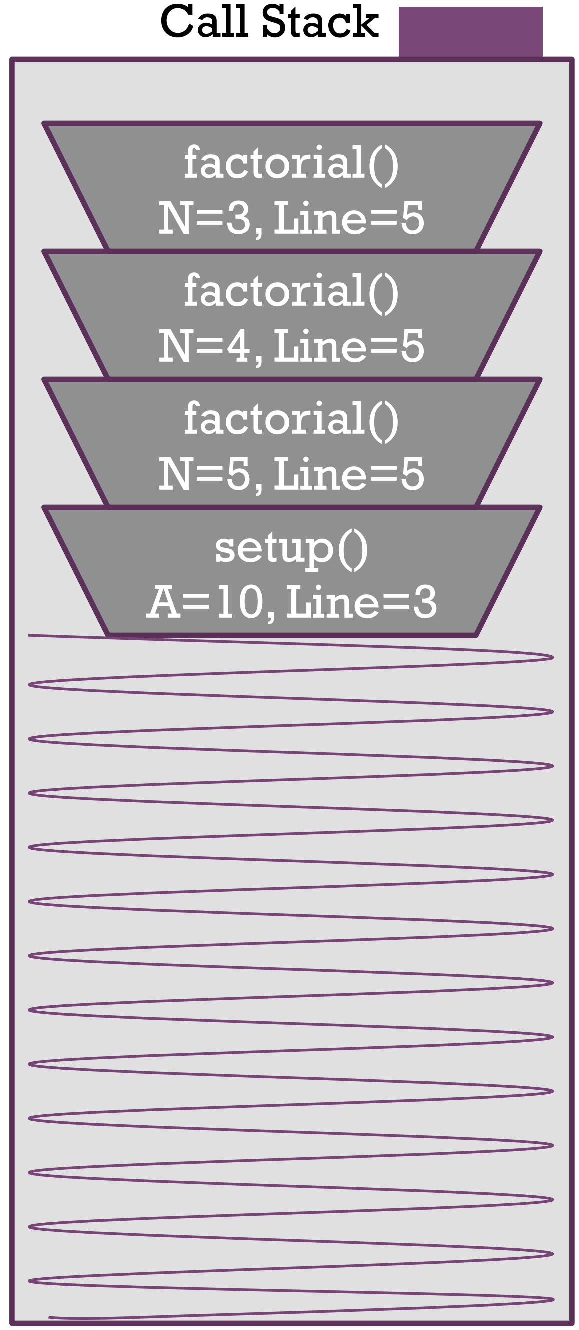
```
1.   void setup() {
2.     int A = 10;
3.     int B = factorial(5);
4.     println( B );
5.   }
```

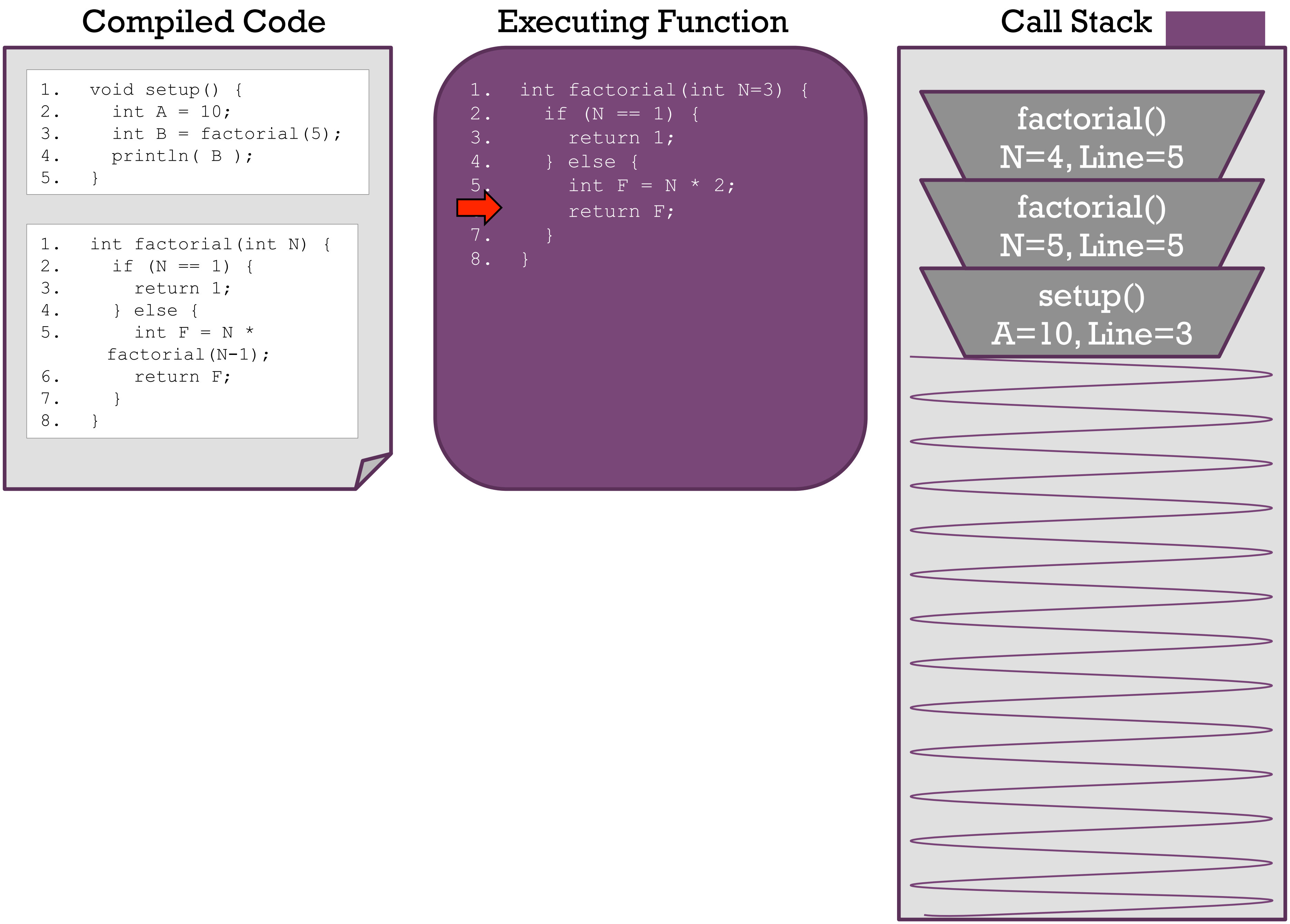
```
1.   int factorial(int N) {
2.     if (N == 1) {
3.       return 1;
4.     } else {
5.       int F = N *
       factorial(N-1);
6.       return F;
7.     }
8.   }
```

Executing Function

```
1.   int factorial(int N=2) {
2.     if (N == 1) {
3.       return 1;
4.     } else {
5.       int F = N * 1;
6.       return F;
7.     }
8.   }
```

Call Stack





Compiled Code
1. void setup() {
2. int A = 10;
3. int B = factorial(5);
4. println(B);
5. }
1. int factorial(int N) {
2. if (N == 1) {
3. return 1;
4. } else {
5. int F = N *
factorial(N-1);
6. return F;
7. }
8. }
Executing Function
1. int factorial(int N=3) {
2. if (N == 1) {
3. return 1;
4. } else {
5. int F = N * 2;
return F;
7. }
8. }
Call Stack
factorial()
N=4, Line=5
factorial()
N=5, Line=5
setup()
A=10, Line=3

Compiled Code

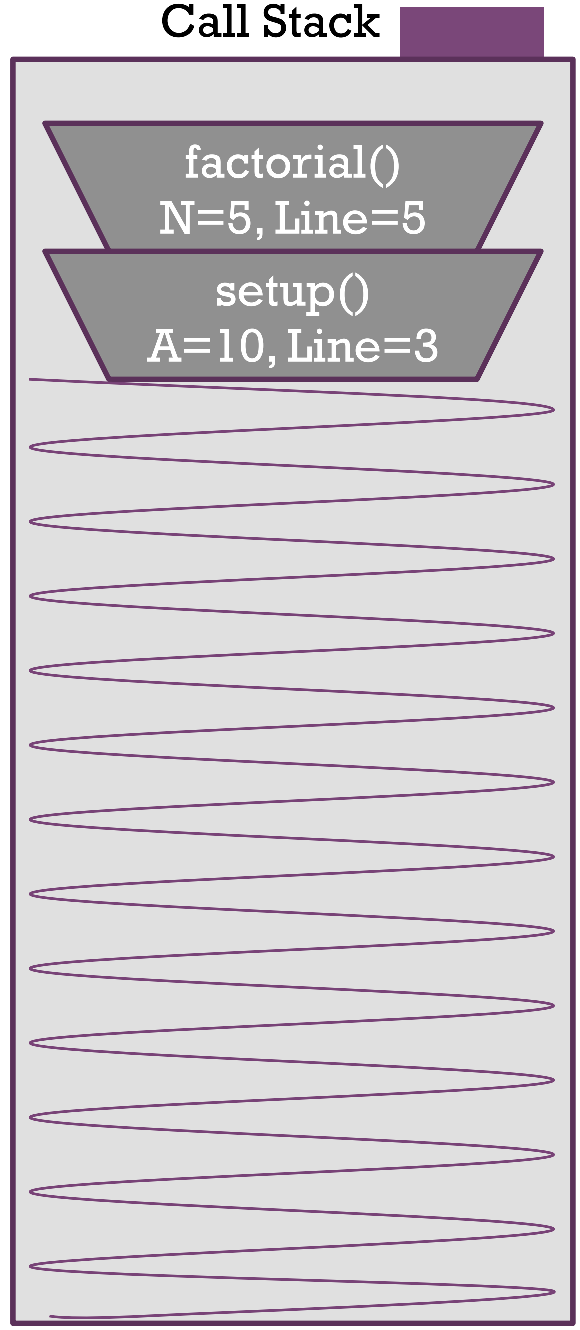
```
1.   void setup() {
2.     int A = 10;
3.     int B = factorial(5);
4.     println( B );
5.   }
```

```
1.   int factorial(int N) {
2.     if (N == 1) {
3.       return 1;
4.     } else {
5.       int F = N *
      factorial(N-1);
6.       return F;
7.     }
8.   }
```

Executing Function

```
1.   int factorial(int N=4) {
2.     if (N == 1) {
3.       return 1;
4.     } else {
5.       int F = N * 6;
6.       return F;
7.     }
8.   }
```

Call Stack



Compiled Code

```
1.   void setup() {
2.     int A = 10;
3.     int B = factorial(5);
4.     println( B );
5.   }
```

```
1.   int factorial(int N) {
2.     if (N == 1) {
3.       return 1;
4.     } else {
5.       int F = N *
       factorial(N-1);
6.       return F;
7.     }
8.   }
```

Executing Function

```
1.   int factorial(int N=5) {
2.     if (N == 1) {
3.       return 1;
4.     } else {
5.       int F = N * 24;
6.       return F;
7.     }
8.   }
```

Call Stack

setup()
A=10, Line=3

Compiled Code

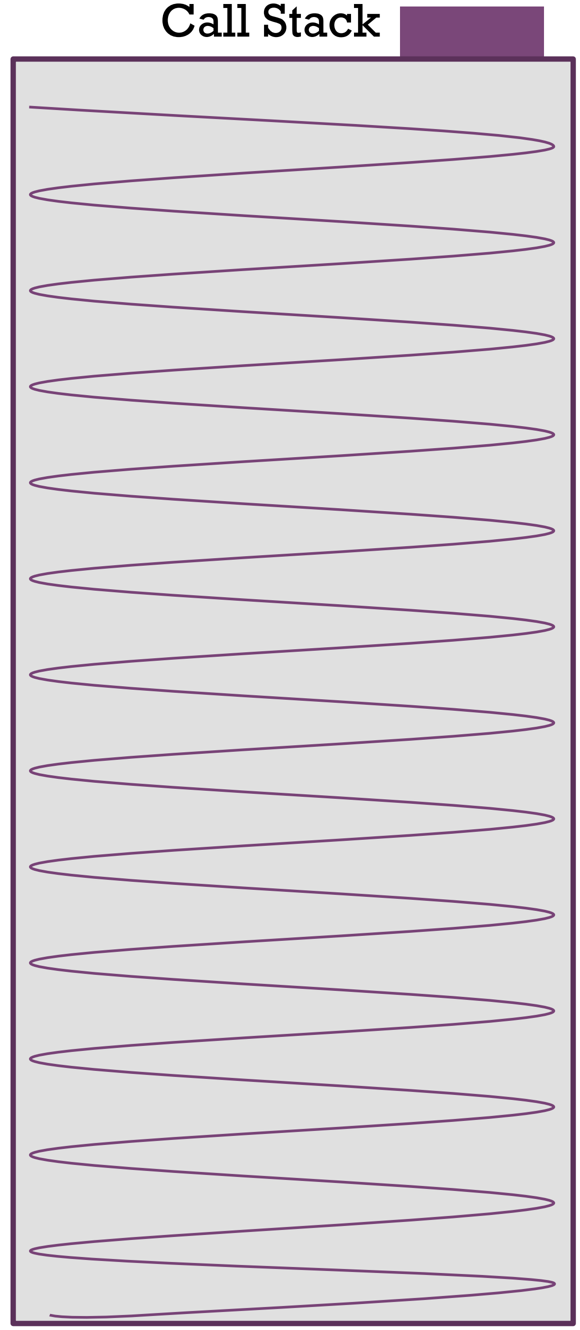
```
void setup() {
  int A = 10;
  int B = factorial(5);
  println( B );
}
```

```
int factorial(int N) {
  if (N == 1) {
    return 1;
  } else {
    int F = N *
  factorial(N-1);
    return F;
  }
}
```

Executing Function

```
void setup() {
  int A = 10;
  int B = 120;
  println( B );
}
```

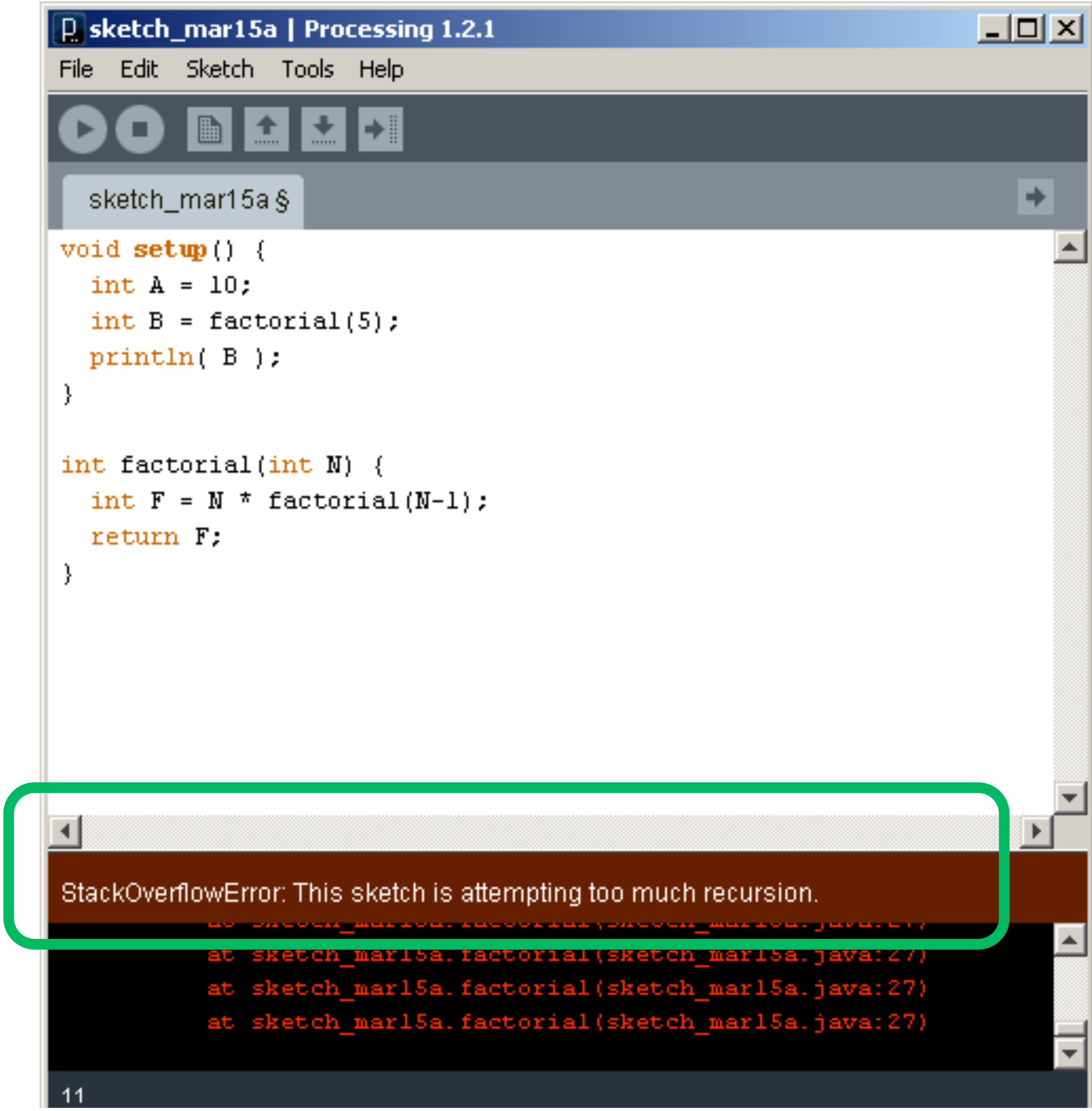
Call Stack



The Call Stack keeps track of …

1. all functions that are suspended, in order
2. the point in the function where execution should resume after the invoked subordinate function returns
3. a snapshot of all variables and values within the scope of the suspended function so these can be restored upon continuing execution

+ What happens if there is no stopping condition, or "base case"?



Recursive Drawing

- Draw a shape, then recursively draw a smaller version of the shape.
- Examples:
  - drawCircles
  - drawSquares

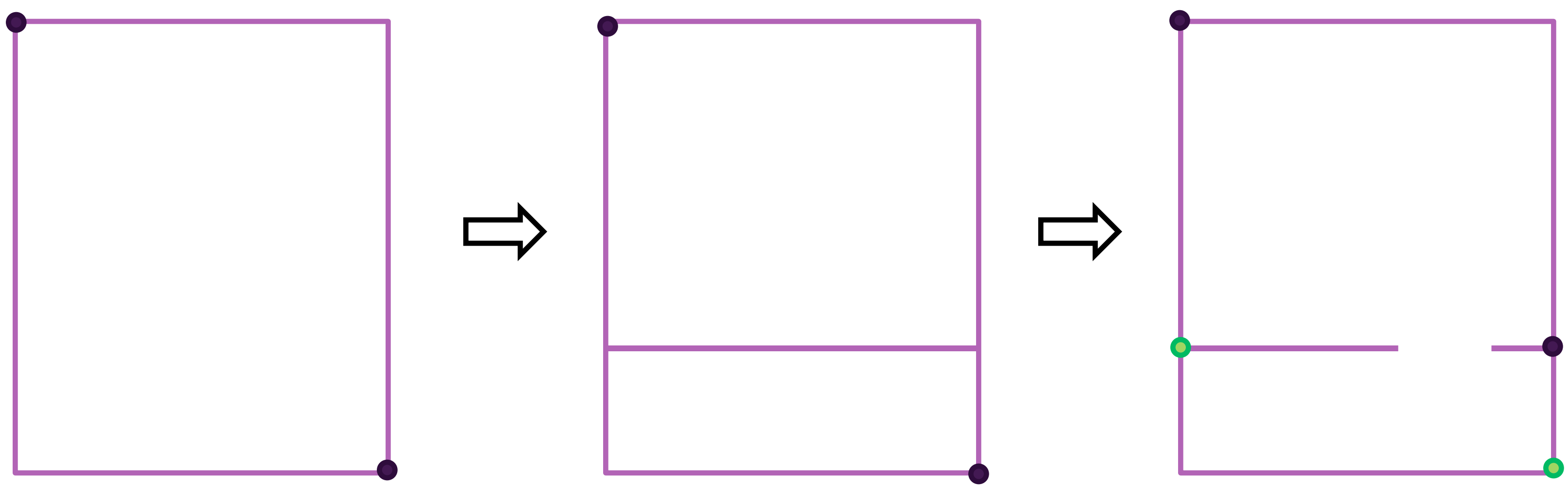
Creating a maze, recursively

1. Start with a rectangular region defined by its upper left and lower right corners

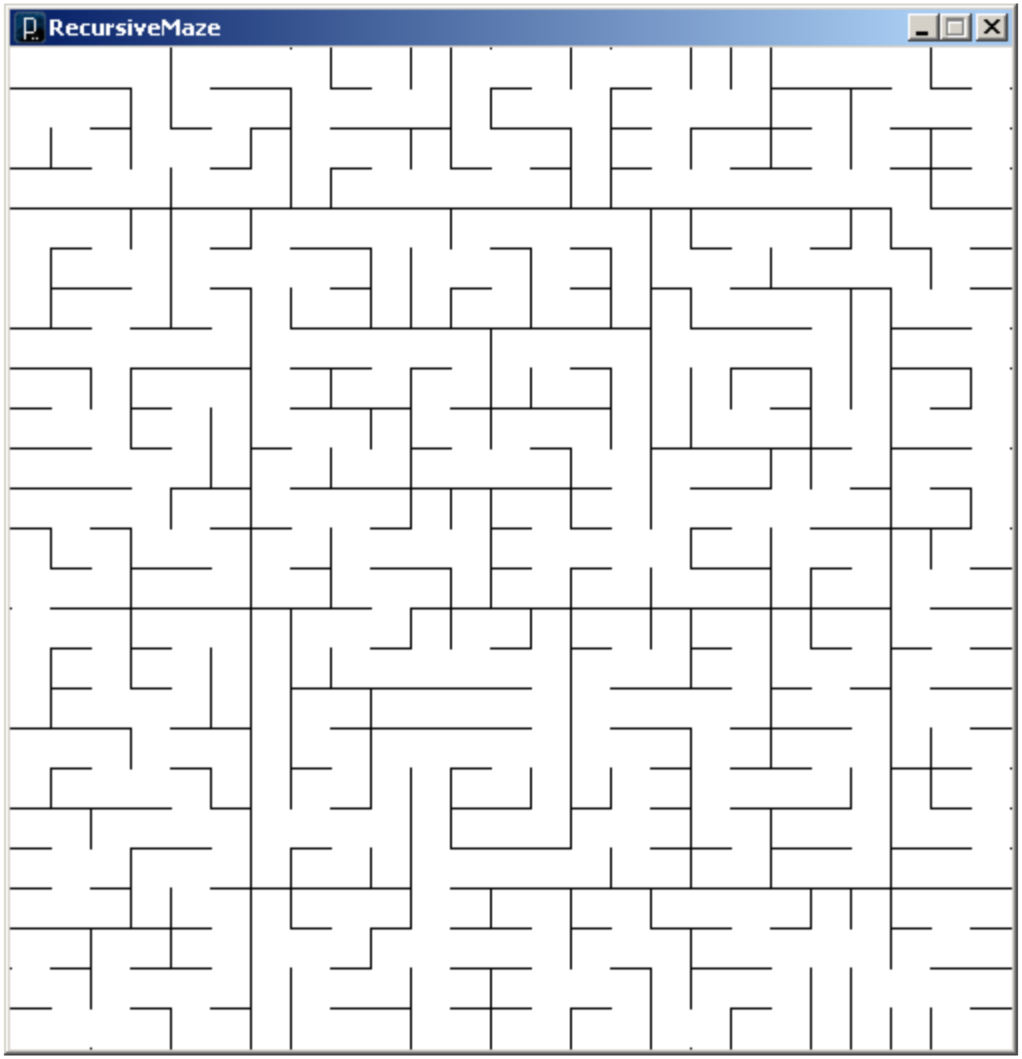
2. Divide the region at a random location through its more narrow dimension

3. Add an opening at a random location

4. Repeat on two rectangular subregions



Inspired by
http://weblog.jamisbuck.org/2011/1/12/maze-generation-recursive-division-algorithm


RecursiveMaze

```
// RecursiveMaze

int N = 25;      // Grid dimension
int gsize = 20; // Grid size

int V = 1;       // Vertical constant
int H = 2;       // Horizontal constant

void setup() {
  // Setup sketch
  size(N*gsize+1, N*gsize+1);
  noLoop();
  background(255);
  stroke(0);

  // Kick off the recursive divide
  // on entire sketch window
  divide(0,0,N,N);
}
```

```
// Determine the direction for dividing
// Stop when too small.
int divDir(int r1, int c1, int r2, int c2) {
  int dr = r2 - r1;           // Deltas
  int dc = c2 - c1;
  if (dr <= 1 || dc <= 1)  // Too small
    return 0;               // No division
  else if (dr < dc)         // Flat and wide
    return V;               // Vertical division
  else                      // Tall and narrow
    return H;               // Horizontal div
}

// Return a random integer in the range
int randomInt(int min, int max) {
  return round(random(min-0.5,max+0.5));
}

// Draw a line on a grid segment
void gridLine(int r1, int c1, int r2, int c2) {
  line(r1*gsize, c1*gsize, r2*gsize, c2*gsize);
}
```

```
// Divide the region given upper left and
// lower right grid corner points

void divide(int r1, int c1, int r2, int c2)
{
  int cr, rr;

  // Get divide direction (V, H or 0)
  int dir = divDir(r1, c1, r2, c2);

  // Divide in vertical direction
  if (dir == V) {

    // Wall and opening locations
    cr = randomInt(c1+1, c2-1);
    rr = randomInt(r1, r2-1);

    // Draw wall
    gridLine(cr,r1,cr,rr);
    gridLine(cr,rr+1,cr,r2);

    // Recursively divide two subregions
    divide(r1,c1,r2,cr);
    divide(r1,cr,r2,c2);

  // Divide in horizontal direction
  } else if (dir == H) {

    // Wall and opening locations
    cr = randomInt(c1, c2-1);
    rr = randomInt(r1+1, r2-1);

    // Draw wall
    gridLine(c1,rr,cr,rr);
    gridLine(cr+1,rr,c2,rr);

    // Recursively divide two subregions
    divide(r1,c1,rr,c2);
    divide(rr,c1,r2,c2);

  // No division. We're done.
  } else {
    return;
  }
}
```