The game of poker

- You are given 5 cards (this is 5-card stud poker)
- The goal is to obtain the best hand you can
- The possible poker hands are (in increasing order):
  - No pair
  - One pair (two cards of the same face)
  - Two pair (two sets of two cards of the same face)
  - Three of a kind (three cards of the same face)
  - Straight (all five cards sequentially – ace is either high or low)
  - Flush (all five cards of the same suit)
  - Full house (a three of a kind of one face and a pair of another face)
  - Four of a kind (four cards of the same face)
  - Straight flush (both a straight and a flush)
  - Royal flush (a straight flush that is 10, J, K, Q, A)

Gambling and Probability

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Poker probability: royal flush

- What is the chance of getting a royal flush?
  - 10, J, Q, K, and A of the same suit
- There are only 4 possible royal flushes
- Cardinality for 5-cards: C(52, 5) = 2,598,960
- Probability = 4/2,598,960 = 0.0000015
  - Or about 1 in 650,000

Poker probability: flush

- What is the chance of a flush?
  - 5 cards of the same suit
- We must do ALL of the following:
  - Pick the suit for the flush: C(4, 1)
  - Pick the 5 cards in that suit: C(13, 5)
- Product rule: C(13, 5)*C(4, 1) = 5148
- Probability = 5148/2,598,960 = 0.00198
  - Or about 1 in 505
- Note that if you don't count straight flushes (and thus royal flushes) as a “flush”, then the number is really 5108

Poker probability: four of a kind

- What is the chance of getting 4 of a kind when dealt 5 cards?
  - 5 cards: C(52, 5) = 2,598,960
- Possible hands that have four of a kind:
  - There are 13 possible four of a kind hands
  - The fifth card can be any of the remaining 48 cards
  - Thus, total is 13*48 = 624
- Probability = 624/2,598,960 = 0.00024
  - Or 1 in 4165

Poker probability: full house

- What is the chance of getting a full house?
  - Three cards of one face and two of another face
- We must do ALL of the following:
  - Pick the face for the three of a kind: C(13, 1)
  - Pick the 3 of the 4 cards to be used: C(4, 3)
  - Pick the face for the pair: C(12, 1)
  - Pick the 2 of the 4 cards of the pair: C(4, 2)
- C(13, 1)*C(4, 3)*C(12, 1)*C(4, 2) = 3744
- Probability = 3744/2,598,960 = 0.00144
  - Or about 1 in 684
Inclusion-exclusion principle

- The possible poker hands are (in increasing order):
  - Nothing
  - One pair
  - Two pair
  - Three of a kind
  - Straight
  - Flush
  - Full house
  - Four of a kind
  - Straight flush
  - Royal flush

Poker hand odds

- The possible poker hands are (in increasing order):
  - Nothing
  - One pair
  - Two pair
  - Three of a kind
  - Straight
  - Flush
  - Full house
  - Four of a kind
  - Straight flush
  - Royal flush

Probability of the union of two events

- Let $E_1$ and $E_2$ be events in sample space $S$
- Then $p(E_1 \cup E_2) = p(E_1) + p(E_2) - p(E_1 \cap E_2)$

Probability of the union of two events

- If you choose a number between 1 and 100, what is the probability that it is divisible by 2 or 5 or both?
- Let $n$ be the number chosen
  - $p(2|n) = 50/100$ (all the even numbers)
  - $p(5|n) = 20/100$
  - $p(2|n)$ and $p(5|n) = p(10|n) = 10/100$
  - $p(2|n)$ or $p(5|n) = p(2|n) + p(5|n) - p(10|n)$
  - $p(2|n)$ or $p(5|n) = 50/100 + 20/100 - 10/100 = 3/5$
When is gambling worth it?

- This is a statistical analysis, not a moral/ethical discussion
- What if you gamble $1, and have a $\frac{1}{2}$ probability to win $10$?
- What if you gamble $1$ and have a $\frac{1}{100}$ probability to win $10$?
- One way to determine if gambling is worth it:
  - probability of winning * payout ≥ amount spent per play

Expected values of gambling

- Gamble $1$, and have a $\frac{1}{2}$ probability to win $10$
  - $(10-1)^{0.5} + (-1)^{0.5} = 4$
- Gamble $1$ and have a $\frac{1}{100}$ probability to win $10$
  - $(10-1)^{0.01} + (-1)^{0.99} = -0.9$
- Another way to determine if gambling is worth it: Expected value > 0

Powerball lottery

- Modern powerball lottery: you pick 5 numbers from 1-55
  - Total possibilities: $C(55,5) = 3,478,761$
- You then pick one number from 1-42 (the powerball)
  - Total possibilities: $C(42,1) = 42$
- You need to do both — apply the product rule,
  - Total possibilities are $3,478,761 \times 42 = 146,107,962$
- While there are many “sub” prizes, the probability for the jackpot is about 1 in 146 million
- If you count in the other prizes, then you will “break even” if the jackpot is $121M

Expected Value

- The expected values of a process with outcomes of values $a_1, a_2, \ldots, a_n$ which occur with probabilities $p_1, p_2, \ldots, p_n$ is:
  \[ \sum_{i=1}^{n} a_i p_i \]

When is lotto worth it?

- In many older lotto games (Pick-6) you have to choose 6 numbers from 1 to 48
  - Total possible choices are $C(48,6) = 12,271,512$
  - Total possible winning numbers is $C(6,6) = 1$
  - Probability of winning is $0.0000000814$
  - Or 1 in 12.3 million
- If you invest $1$ per ticket, it is only statistically worth it if the payout is $> 12.3$ million

Blackjack

- You are initially dealt two cards
  - 10, J, Q and K all count as 10
  - Ace is EITHER 1 or 11 (player’s choice)
- You can opt to receive more cards (a “hit”)
- You want to get as close to 21 as you can
  - If you go over, you lose (a “bust”)
- You play against the house
  - If the house has a higher score than you, then you lose
Blackjack table

Blackjack probabilities

- Getting 21 on the first two cards is called a blackjack
  - Or a "natural 21"
- Assume there is only 1 deck of cards
- Possible blackjack blackjack hands:
  - First card is an A, second card is a 10, J, Q, or K
    - 4/52 for Ace, 16/51 for the ten card
    - (4/16)(10/51) = 0.0244 (or about 1 in 41)
  - First card is a 10, J, Q, or K, second card is an A
    - 16/52 for the ten card, 4/51 for Ace
    - (16/52)(4/51) = 0.0244 (or about 1 in 41)
- Total chance of getting a blackjack is the sum of the two:
  - P = 0.0488, or about 1 in 21
  - More specifically, it’s 1 in 20.72

Blackjack probabilities

- Assume there is an infinite deck of cards
- Possible blackjack blackjack hands:
  - First card is an A, second card is a 10, J, Q, or K
    - 4/52 for Ace, 16/52 for second part
    - (4/52)(10/52) = 0.0236 (or about 1 in 42)
  - First card is a 10, J, Q, or K, second card is an A
    - 16/52 for first part, 4/52 for Ace
    - (16/52)(4/52) = 0.0236 (or about 1 in 42)
- Total chance of getting a blackjack is the sum:
  - P = 0.0473, or about 1 in 21
  - More specifically, it’s 1 in 21.13 (vs. 20.72)

- In reality, most casinos use “shoes” of 6-8 decks for this reason
  - It slightly lowers the player’s chances of getting a blackjack
  - And prevents people from counting the cards...

Counting cards and Continuous Shuffling Machines (CSMs)

- Counting cards means keeping track of which cards have been dealt, and how that modifies the chances
- After cards are discarded, they are added to the continuous shuffling machine
- Many blackjack players refuse to play at a casino with one
  - So they aren’t used as much as casinos would like

So always use a single deck, right?

- Most people think that a single-deck blackjack table is better, as the player’s odds increase
  - And you can try to count the cards
- Normal rules have a 3:2 payout for blackjack
  - If you bet $100, you get your $100 back plus 3/2 * $100, or $150 additional
- Most single-deck tables have a 6:5 payout
  - You get your $100 back plus 6/5 * $100 or $120 additional
  - The expected value of the game is lowered
  - This OUTWEIGHS the benefit of the single deck!
  - And the benefit of counting the cards
  - Remember, the house always wins
Buying (blackjack) insurance

- If the dealer’s visible card is an Ace, the player can buy insurance against the dealer having a blackjack
  - There are then two bets going: the original bet and the insurance bet
  - If the dealer has blackjack, you lose your original bet, but your insurance bet pays 2-to-1.
    - So you get twice what you paid in insurance back
    - Note that if the player also has a blackjack, it’s a “push”
  - If the dealer does not have blackjack, you lose your insurance bet, but your original bet proceeds normal.
- Is this insurance worth it?

Why counting cards doesn't work well...

- If you make two or three mistakes an hour, you lose any advantage
  - And, in fact, cause a disadvantage!
- You lose lots of money learning to count cards
- Then, once you can do so, you are banned from the casinos

So why is Blackjack so popular?

- Although the casino has the upper hand, the odds are much closer to 50-50 than with other games
  - Players following strategy will lose less than 1% on average luck
- Notable exceptions are games that you are not playing against the house – i.e., poker

Blackjack Strategy Chart

Roulette

- A wheel with 38 spots is spun
  - Spots are numbered 1-36, 0, and 00
  - European casinos don't have the 00
- A ball drops into one of the 38 spots
- A bet is placed as to which spot or spots the ball will fall into
  - Money is then paid out if the ball lands in the spot(s) you bet upon
The Roulette table

• Bets can be placed on:
  – A single number 1/38
  – Two numbers 2/38
  – Four numbers 4/38
  – All even numbers 18/38
  – All odd numbers 18/38
  – The first 18 nums 18/38
  – Red numbers 18/38

Probability:

1/38
2/38
3/38
4/38
18/38
18/38
18/38
18/38

Payout:

36x
18x
9x
2x
2x
2x

Roulette

• It has been proven that no advantageous strategies exist
  • Including:
    – Learning the wheel’s biases
    • Casinos regularly balance their Roulette wheels
    – Using lasers (yes, lasers) to check the wheel’s spin
    • What casino will let you set up a laser inside to beat the house?

Roulette

• Martingale betting strategy
  – Where you double your (outside) bet each time (thus making up for all previous losses)
  – It still won’t work!
  – You can’t double your money forever
    • It could easily take 50 times to achieve a final win
    • If you start with $1, then you must put in $1*2^{50} = $1,125,899,906,842,624 to win this way!
    • That’s 1 quadrillion