Uninformed Search

Chapter 3

Some material adopted
from notes and slides by
Marie desJardins and
Charles R. Dyer

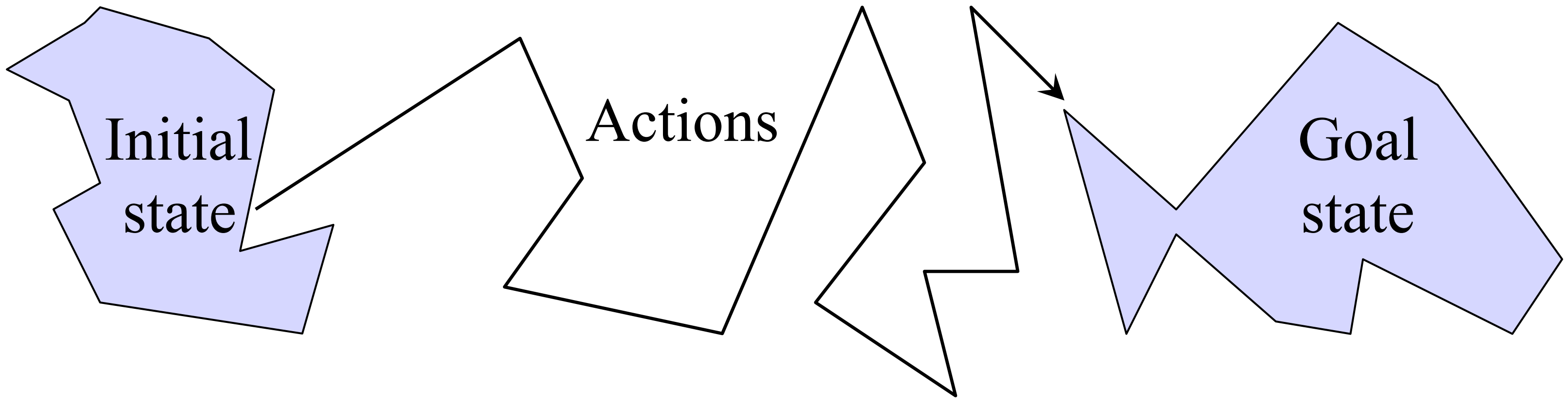
Today's class

- Goal-based agents
- Representing states and operators
- Example problems
- Generic state-space search algorithm
- Specific algorithms
  - Breadth-first search
  - Depth-first search
  - Uniform cost search
  - Depth-first iterative deepening
- Example problems revisited

Building goal-based agents

To build a goal-based agent we need to answer the following questions:

- What is the goal to be achieved?
- What are the actions?
- What is the representation?
    - E.g., what *relevant* information is necessary to encode in order to describe the state of the world, describe the available transitions, and solve the problem?)



What is the goal to be achieved?

- Could describe a situation we want to achieve, a set of properties that we want to hold, etc.
- Requires defining a **“goal test”** so that we know what it means to have achieved/satisfied our goal.
- This is a hard question that is rarely tackled in AI, usually assuming that the system designer or user will specify the goal to be achieved.
- Certainly psychologists and motivational speakers always stress the importance of people establishing clear goals for themselves as the first step towards solving a problem.

What are the actions?

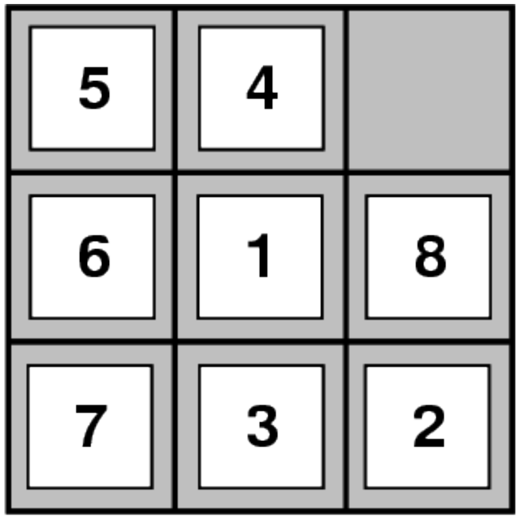
- Characterize the **primitive actions** or events that are available for making changes in the world in order to achieve a goal.
- **Deterministic** world: no uncertainty in an action's effects. Given an action (a.k.a. **operator** or move) and a description of the **current world state**, the action completely specifies
  - whether that action *can* be applied to the current world (i.e., is it applicable and legal), and
  - what the exact state of the world will be after the action is performed in the current world (i.e., no need for "history" information to compute what the new world looks like).

What are the actions? (cont’d)

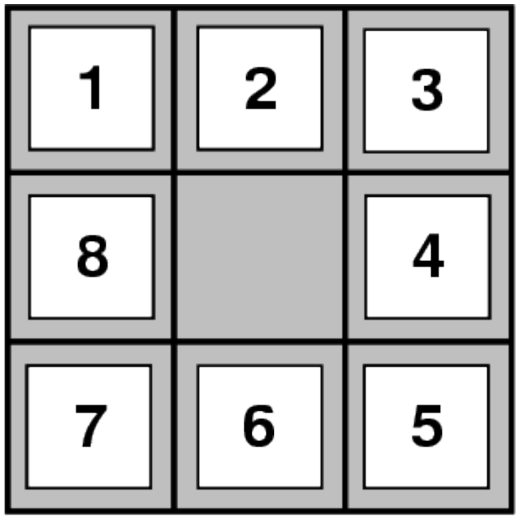
- Note also that actions in this framework can all be considered as **discrete events** that occur at an **instant of time**.
  - For example, if “Mary is in class” and then performs the action “go home,” then in the next situation she is “at home.” There is no representation of a point in time where she is neither in class nor at home (i.e., in the state of “going home”).
- The actions are largely problem-specific and determined (intelligently ;-)) by the system designer.
- There usually are multiple action sets for solving the same problem.
- Let’s look an example…

8-Puzzle

Given an initial configuration of 8 numbered tiles on a 3 x 3 board, move the tiles in such a way so as to produce a desired goal configuration of the tiles.



Start State



Goal State

Representing actions

- The number of actions / operators depends on the **representation** used in describing a state.
  - In the 8-puzzle, we could specify 4 possible moves for each of the 8 tiles, resulting in a total of **4*8=32 operators**.
  - On the other hand, we could specify four moves for the "blank" square and we would only need **4 operators**.
- Representational shift can greatly simplify a problem!

Representing states

- What information is necessary to encode about the world to sufficiently describe all relevant aspects to solving the goal? That is, what knowledge needs to be represented in a state description to adequately describe the current state or situation of the world?
- The **size of a problem** is usually described in terms of the **number of states** that are possible.
  - The 8-puzzle has 181,440 states.
  - Tic-Tac-Toe has about 3^9 states.
  - Rubik's Cube has about 10^{19} states.
  - Checkers has about 10^{40} states.
  - Chess has about 10^{120} states in a typical game.

Closed World Assumption

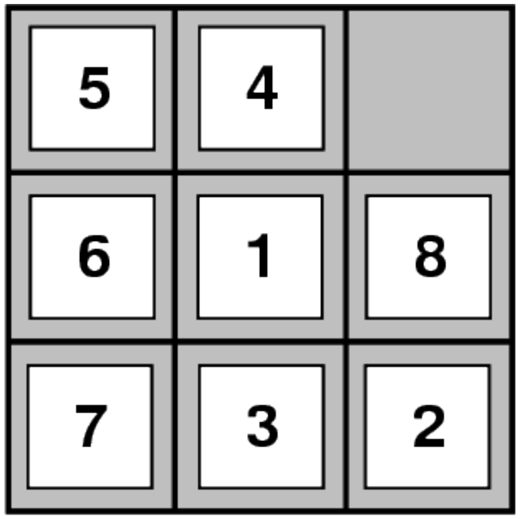
- We will generally use the **Closed World Assumption**.
- All necessary information about a problem domain is available in each percept so that <u>each state is a complete description of the world</u>.
- There is no incomplete information at any point in time.

Some example problems

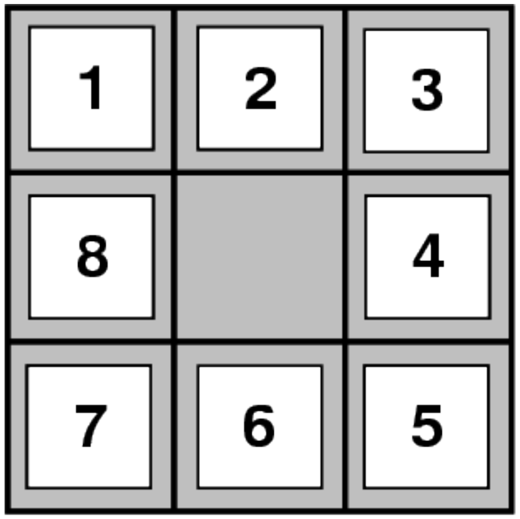
- Toy problems and micro-worlds
  - 8-Puzzle
  - Missionaries and Cannibals
  - Cryptarithmetic
  - Remove 5 Sticks
  - Water Jug Problem
- Real-world problems

8-Puzzle

Given an initial configuration of 8 numbered tiles on a 3 x 3 board, move the tiles in such a way so as to produce a desired goal configuration of the tiles.



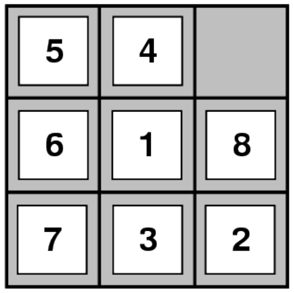
Start State



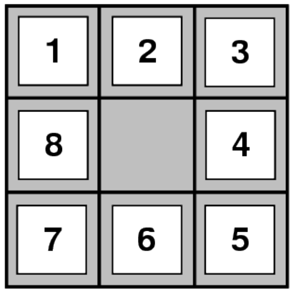
Goal State

8-Puzzle

- **State Representation:** 3 x 3 array configuration of the tiles on the board.
- **Operators:** Move Blank Square Left, Right, Up or Down.
  - This is a more efficient encoding of the operators than one in which each of four possible moves for each of the 8 distinct tiles is used.
- **Initial State:** A particular configuration of the board.
- **Goal:** A particular configuration of the board.



Start State



Goal State

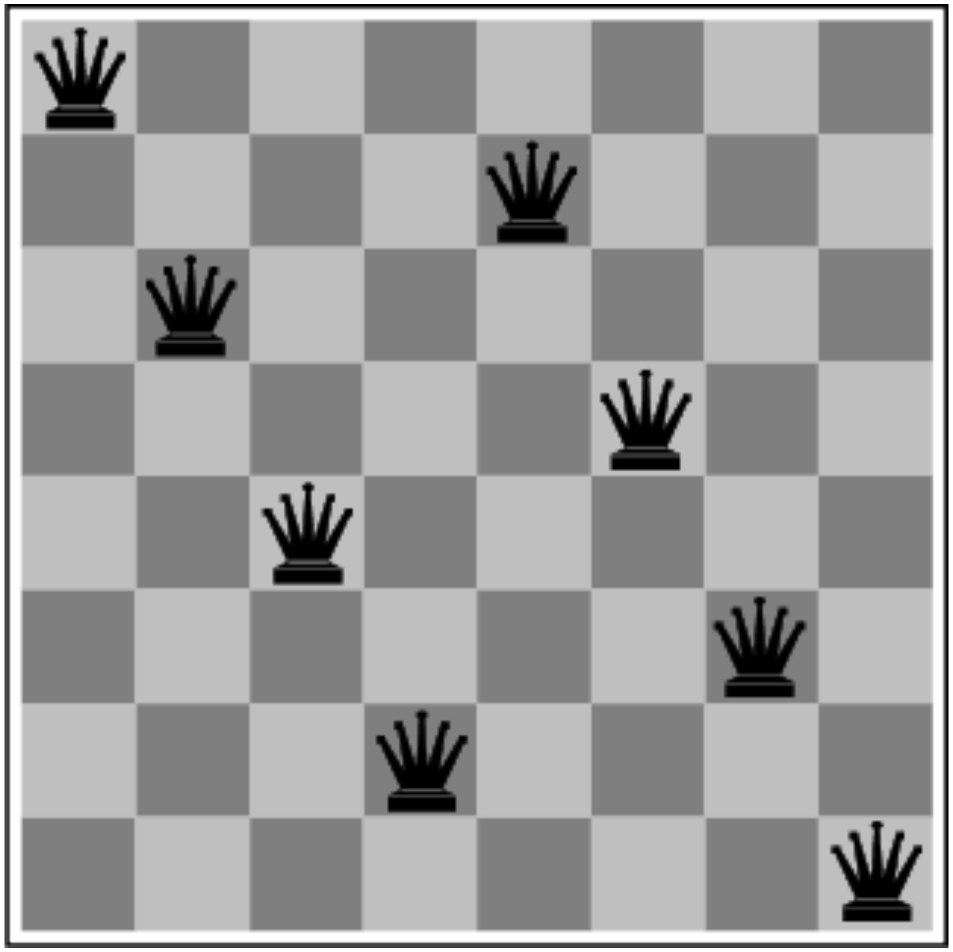
The 8-Queens Problem

State Representation: ?

Initial State: ?

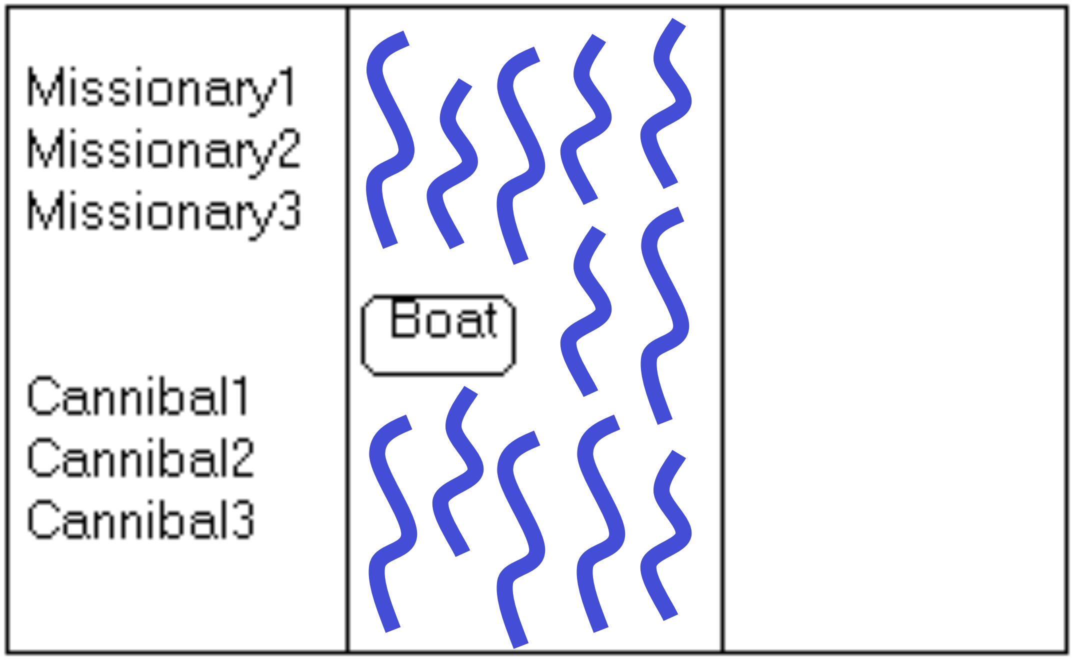
Operators: ?

Goal: Place eight queens on a chessboard such that no queen attacks any other!



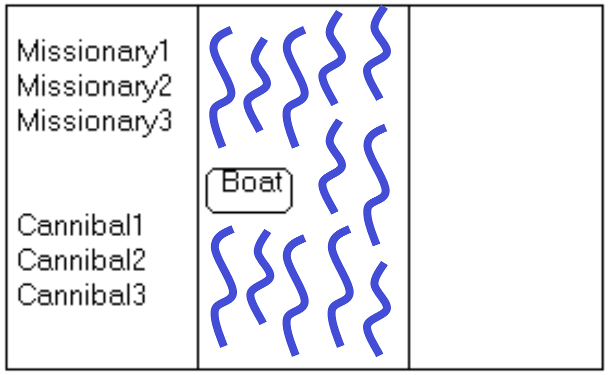
Missionaries and Cannibals

Three missionaries and three cannibals wish to cross the river. They have a small boat that will carry up to two people. Everyone can navigate the boat. If at any time the Cannibals outnumber the Missionaries on either bank of the river, they will eat the Missionaries. Find the smallest number of crossings that will allow everyone to cross the river safely.



Missionaries and Cannibals

- **Goal**: Move all the missionaries and cannibals across the river.
- **Constraint:** Missionaries can never be outnumbered by cannibals on either side of river, or else the missionaries are killed.
- **State:** configuration of missionaries and cannibals and boat on each side of river.
- **Initial State:** 3 missionaries, 3 cannibals and the boat are on the near bank
- **Operators:** Move boat containing some set of occupants across the river (in either direction) to the other side.



3 Missionaries and 3 Cannibals wish to cross the river. They have a boat that will carry two people. Everyone can navigatethe boat. If at any time the Cannibals outnumber the missionaries on either bank of the river, they will eat the Missionaries. Find the smallest number of crossings that will allow everyone to cross the river safely.
The problem can be solved in 11 moves. But people rarely get the optimal solution, because the MC problem contains a 'tricky' state at the end, where two people move back across the river.

Missionaries and Cannibals Solution

	Near side		***Far side***	
0 Initial setup:	MMMCCC	B		-
1 Two cannibals cross over:	MMMC		B	CC
2 One comes back:	MMMCC	B		C
3 Two cannibals go over again:	MMM		B	CCC
4 One comes back:	MMMC	B		CC
5 Two missionaries cross:	MC		B	MMCC
6 A missionary & cannibal return:	MMCC	B		MC
7 Two missionaries cross again:	CC		B	MMMC
8 A cannibal returns:	CCC	B		MMM
9 Two cannibals cross:	C		B	MMMCC
10 One returns:	CC	B		MMMC
11 And brings over the third:	-		B	MMMCCC

Cryptarithmetic

- Find an assignment of digits (0, ..., 9) to letters so that a given arithmetic expression is true. examples: SEND + MORE = MONEY and

```
 FORTY     Solution:  29786
+  TEN                  850
+  TEN                  850
 -----                -----
 SIXTY                31486
F=2, O=9, R=7, etc.
```

Cryptarithmetic

- **State:** mapping from letters to digits
- **Initial State:** empty mapping
- **Operators:** assign a digit to a letter
- **Goal Test:** whether the expression is true given the <u>complete</u> mapping

Note: In this problem, the solution is NOT a sequence of actions that transforms the initial state into the goal state; rather, the solution is a goal node that includes an assignment of a digit to each letter in the given problem.

Find an assignment of digits to letters so that a given arithmetic expression is true. examples: SEND + MORE = MONEY and

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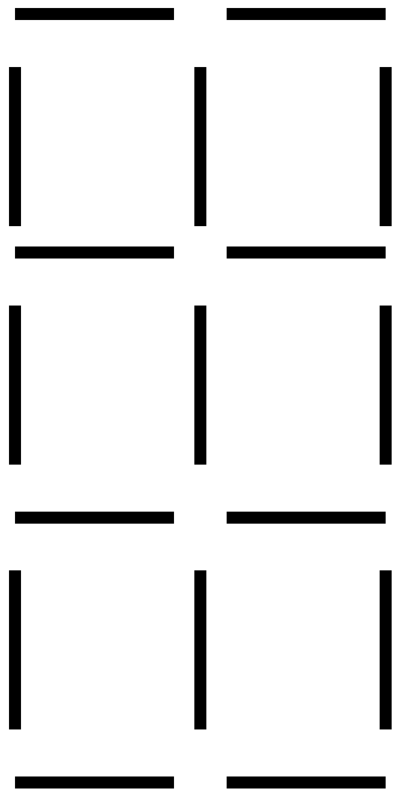
Remove 5 Sticks

Given the following configuration of sticks, remove exactly 5 sticks in such a way that the remaining configuration forms exactly 3 squares.

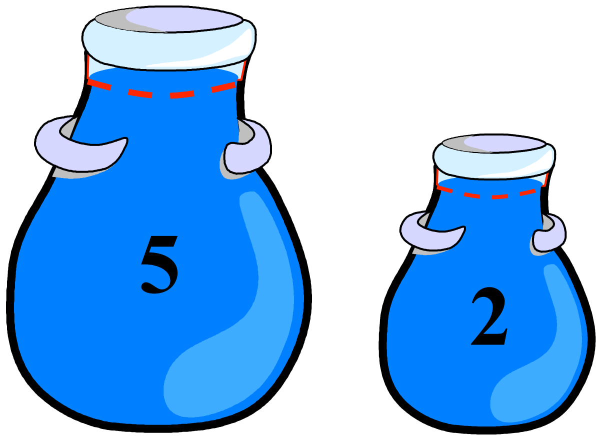


- **State:** ?
- **Initial State:** ?
- **Operators:** ?
- **Goal Test:** ?

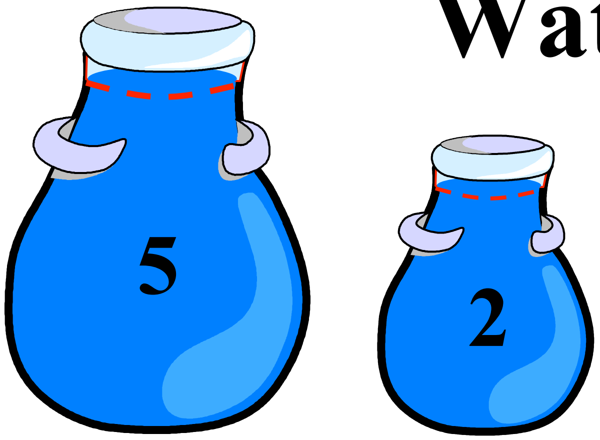
Water Jug Problem

Given a full 5-gallon jug and a full 2-gallon jug, fill the 2-gallon jug with exactly one gallon of water.

- **State:** ?
- **Initial State:** ?
- **Operators:** ?
- **Goal State:** ?



Water Jug Problem



- State = (x,y), where x is the number of gallons of water in the 5-gallon jug and y is # of gallons in the 2-gallon jug
- Initial State = (5,2)
- Goal State = (*,1), where * means any amount

Operator table

Name	Cond.	Transition	Effect
Empty5	–	$(x,y) \rightarrow (0,y)$	Empty 5-gal. jug
Empty2	–	$(x,y) \rightarrow (x,0)$	Empty 2-gal. jug
2to5	$x \leq 3$	$(x,2) \rightarrow (x+2,0)$	Pour 2-gal. into 5-gal.
5to2	$x \geq 2$	$(x,0) \rightarrow (x-2,2)$	Pour 5-gal. into 2-gal.
5to2part	$y < 2$	$(1,y) \rightarrow (0,y+1)$	Pour partial 5-gal. into 2-gal.

Some more real-world problems

- Route finding
- Touring (traveling salesman)
- Logistics
- VLSI layout
- Robot navigation
- Learning

Knowledge representation issues

- What's in a state ?
  - Is the color of the boat relevant to solving the Missionaries and Cannibals problem? Is sunspot activity relevant to predicting the stock market? What to represent is a very hard problem that is usually left to the system designer to specify.
- What **level of abstraction** or detail to describe the world.
  - Too fine-grained and we'll "miss the forest for the trees." Too coarse-grained and we'll miss critical details for solving the problem.
- The number of states depends on the representation and level of abstraction chosen.
  - In the Remove-5-Sticks problem, if we represent the individual sticks, then there are 17-choose-5 possible ways of removing 5 sticks.
  - On the other hand, if we represent the "squares" defined by 4 sticks, then there are 6 squares initially and we must remove 3 squares, so only 6-choose-3 ways of removing 3 squares.

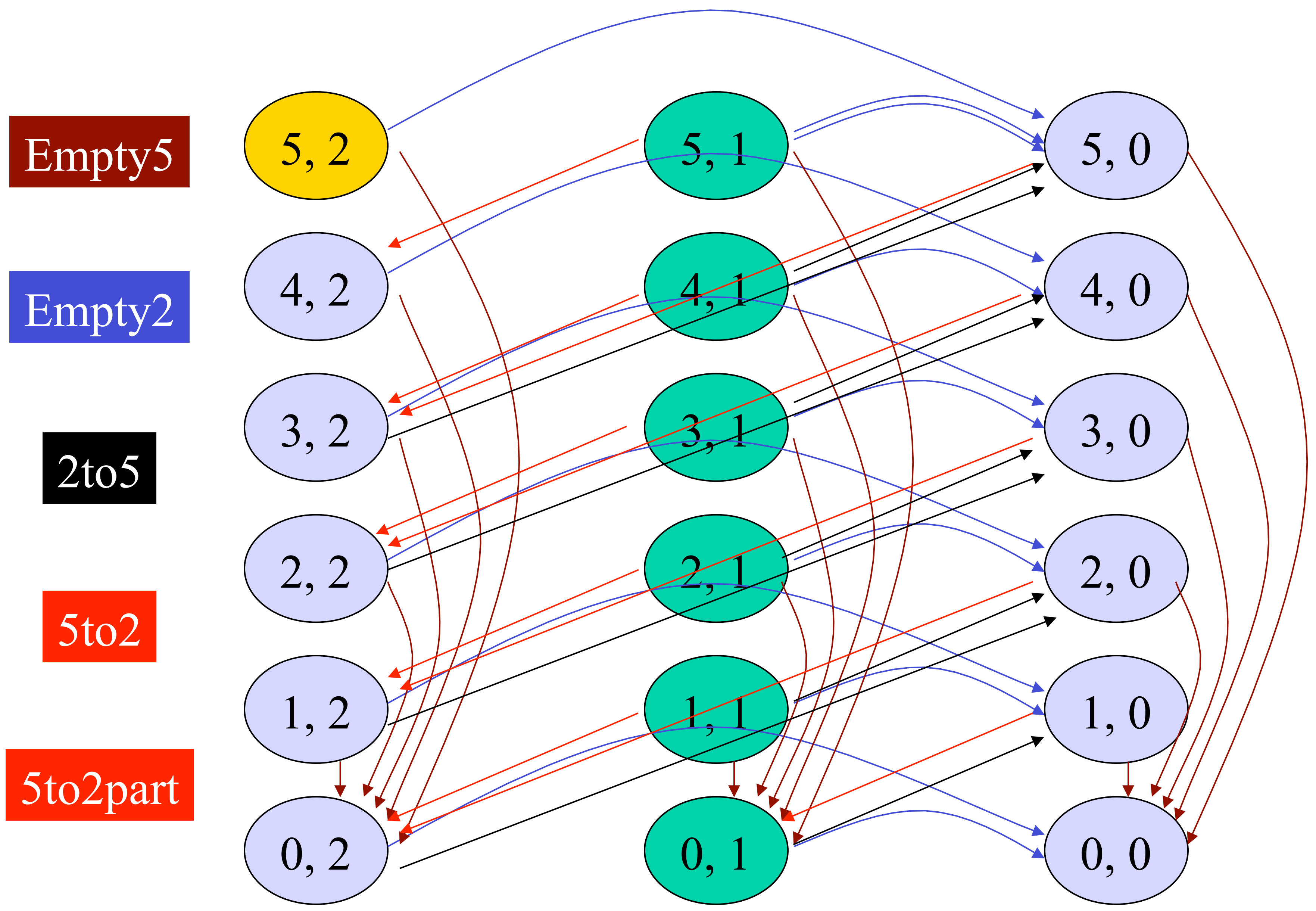
Formalizing search in a state space

- A state space is a **graph** (V, E) where V is a set of **nodes** and E is a set of **arcs**, and each arc is directed from a node to another node
- Each **node** is a data structure that contains a state description plus other information such as the parent of the node, the name of the operator that generated the node from that parent, and other bookkeeping data
- Each **arc** corresponds to an instance of one of the operators. When the operator is applied to the state associated with the arc's source node, then the resulting state is the state associated with the arc's destination node

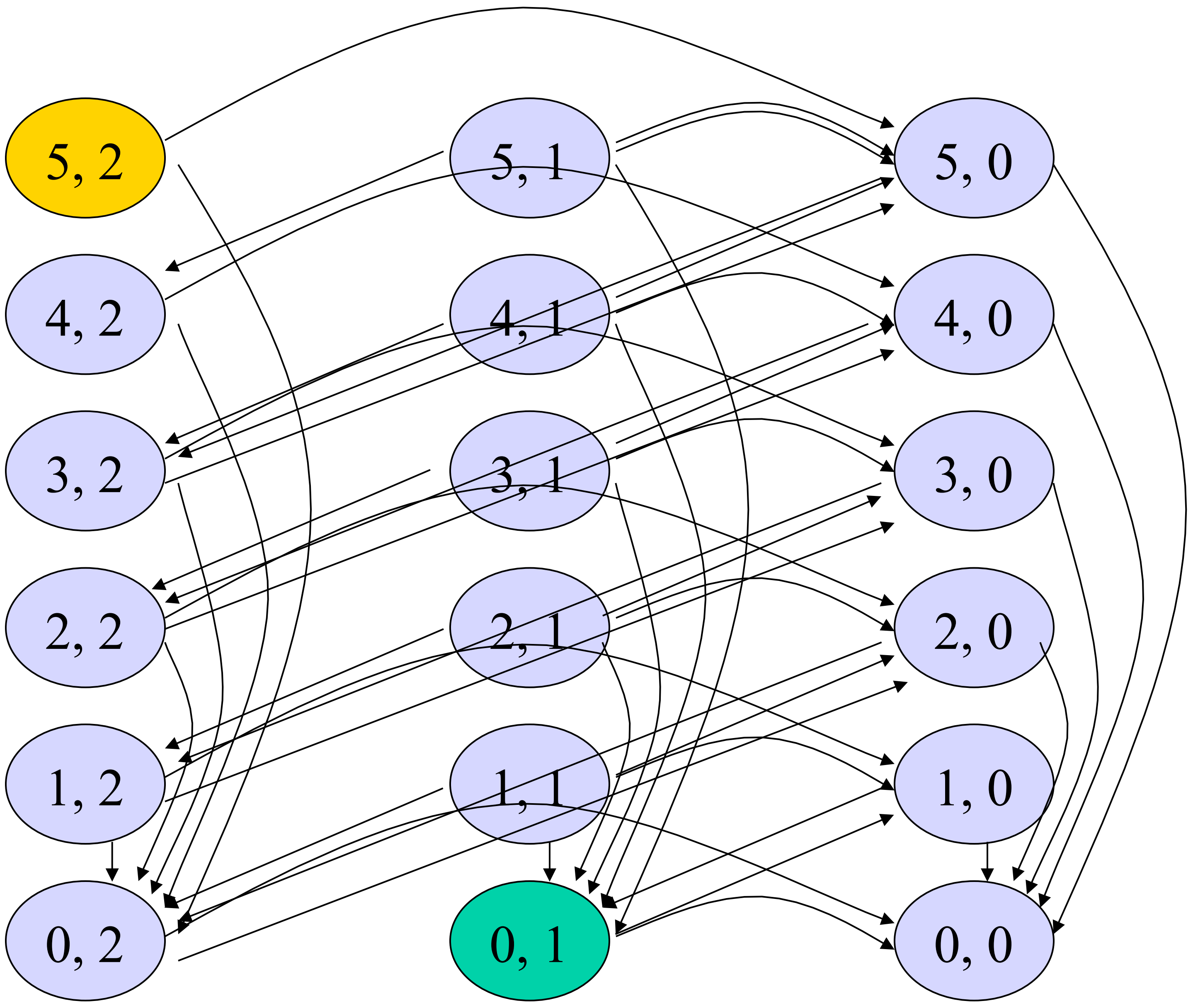
Formalizing search II

- Each arc has a fixed, positive **cost** associated with it corresponding to the cost of the operator.
- Each node has a set of **successor nodes** corresponding to all of the legal operators that can be applied at the source node's state.
  - The process of expanding a node means to generate all of the successor nodes and add them and their associated arcs to the state-space graph
- One or more nodes are designated as **start nodes.**
- A **goal test** predicate is applied to a state to determine if its associated node is a goal node.

Water jug state space



Water jug solution



Formalizing search III

- A **solution** is a sequence of operators that is associated with a path in a state space from a start node to a goal node.
- The **cost of a solution** is the sum of the arc costs on the solution path.
  - If all arcs have the same (unit) cost, then the solution cost is just the length of the solution (number of steps / state transitions)

Formalizing search IV

- **State-space search** is the process of searching through a state space for a solution by **making explicit** a sufficient portion of an **implicit** state-space graph to find a goal node.
  - For large state spaces, it isn't practical to represent the whole space.
  - Initially V={S}, where S is the start node; when S is expanded, its successors are generated and those nodes are added to V and the associated arcs are added to E. This process continues until a goal node is found.
- Each node implicitly or explicitly represents a partial solution path (and cost of the partial solution path) from the start node to the given node.
  - In general, from this node there are many possible paths (and therefore solutions) that have this partial path as a prefix.

State-space search algorithm

```
function general-search (problem, QUEUEING-FUNCTION)
;; problem describes the start state, operators, goal test, and operator costs
;; queueing-function is a comparator function that ranks two states
;; general-search returns either a goal node or failure
nodes = MAKE-QUEUE(MAKE-NODE(problem.INITIAL-STATE))
  loop
     if EMPTY(nodes) then return "failure"
     node = REMOVE-FRONT(nodes)
     if problem.GOAL-TEST(node.STATE) succeeds
        then return node
     nodes = QUEUEING-FUNCTION(nodes, EXPAND(node,
                problem.OPERATORS))
 end
  ;; Note: The goal test is NOT done when nodes are generated
  ;; Note: This algorithm does not detect loops
```

Key procedures to be defined

- EXPAND
  - Generate all successor nodes of a given node
- GOAL-TEST
  - Test if state satisfies all goal conditions
- QUEUEING-FUNCTION
  - Used to maintain a ranked list of nodes that are candidates for expansion

Bookkeeping

- Typical node data structure includes:
  - State at this node
  - Parent node
  - Operator applied to get to this node
  - Depth of this node (number of operator applications since initial state)
  - Cost of the path (sum of each operator application so far)

Some issues

- Search process constructs a search tree, where
  - **root** is the initial state and
  - **leaf nodes** are nodes
    - not yet expanded (i.e., they are in the list "nodes") or
    - having no successors (i.e., they're "deadends" because no operators were applicable and yet they are not goals)
- Search tree may be infinite because of loops even if state space is small
- Return a path or a node depending on problem.
  - E.g., in cryptarithmetic return a node; in 8-puzzle return a path
- Changing definition of the QUEUEING-FUNCTION leads to different search strategies

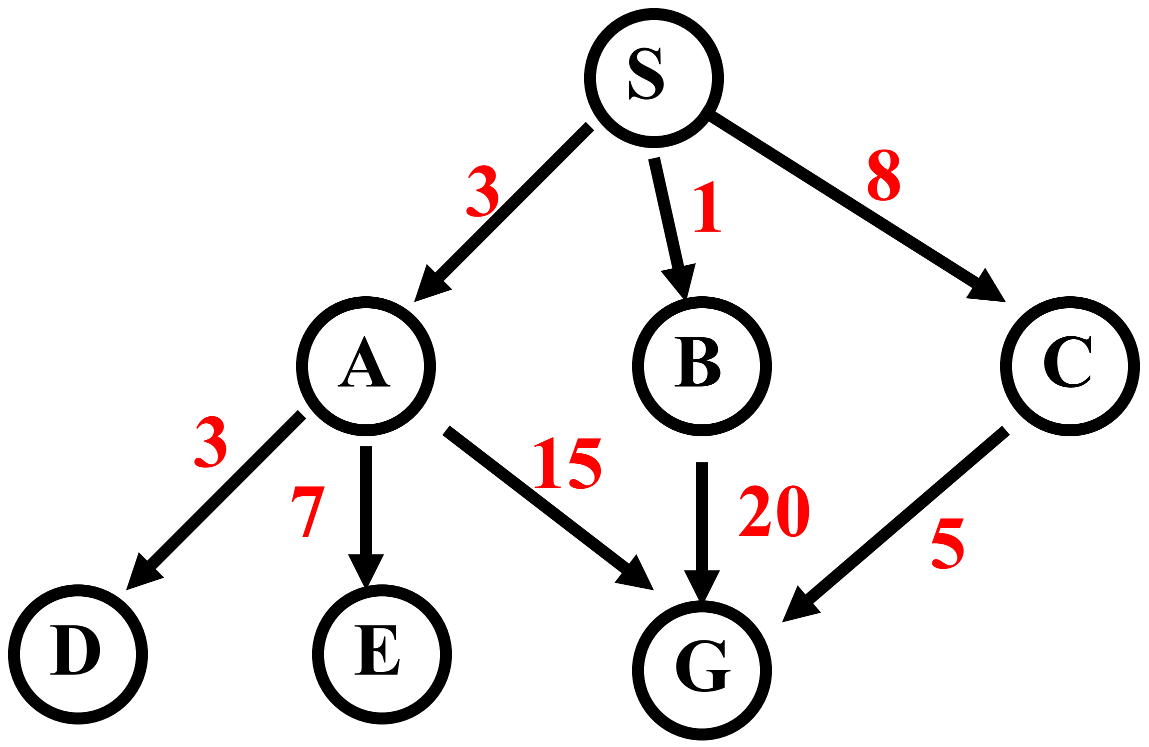
Evaluating Search Strategies

- **Completeness**
  - Guarantees finding a solution whenever one exists
- **Time complexity**
  - How long (worst or average case) does it take to find a solution? Usually measured in terms of the number of nodes expanded
- **Space complexity**
  - How much space is used by the algorithm? Usually measured in terms of the maximum size of the "nodes" list during the search
- **Optimality/Admissibility**
  - If a solution is found, is it guaranteed to be an optimal one? That is, is it the one with minimum cost?

Uninformed vs. informed search

- **Uninformed search strategies**
  - Also known as “blind search,” uninformed search strategies use no information about the likely “direction” of the goal node(s)
  - Uninformed search methods: Breadth-first, depth-first, depth-limited, uniform-cost, depth-first iterative deepening, bidirectional
- **Informed search strategies**
  - Also known as “heuristic search,” informed search strategies use information about the domain to (try to) (usually) head in the general direction of the goal node(s)
  - Informed search methods: Hill climbing, best-first, greedy search, beam search, A, A*

Example for illustrating uninformed search strategies



Uninformed Search Methods

Breadth-First

- Enqueue nodes in **FIFO** (first-in, first-out) order.
- **Complete**
- **Optimal** (i.e., admissible) if all operators have the same cost. Otherwise, not optimal but finds solution with shortest path length.
- **Exponential time and space complexity**, $O(b^d)$, where d is the depth of the solution and b is the branching factor (i.e., number of children) at each node
- Will take a **long time to find solutions** with a large number of steps because must look at all shorter length possibilities first
  - A complete search tree of depth d where each non-leaf node has b children, has a total of $1 + b + b^2 + ... + b^d = (b^{(d+1)} - 1)/(b-1)$ nodes
  - For a complete search tree of depth 12, where every node at depths 0, ..., 11 has 10 children and every node at depth 12 has 0 children, there are $1 + 10 + 100 + 1000 + ... + 10^{12} = (10^{13} - 1)/9 = O(10^{12})$ nodes in the complete search tree. If BFS expands 1000 nodes/sec and each node uses 100 bytes of storage, then BFS will take 35 years to run in the worst case, and it will use 111 terabytes of memory!

Depth-First (DFS)

- Enqueue nodes in **LIFO** (last-in, first-out) order. That is, use a stack data structure to order nodes.
- **May not terminate** without a "depth bound," i.e., cutting off search below a fixed depth D ("depth-limited search")
- **Not complete** (with or without cycle detection, and with or without a cutoff depth)
- **Exponential time**, O(b^d), but only **linear space**, O(bd)
- Can find **long solutions quickly** if lucky (and **short solutions slowly** if unlucky!)
- When search hits a dead-end, can only back up one level at a time even if the "problem" occurs because of a bad operator choice near the top of the tree. Hence, only does "chronological backtracking"

Uniform-Cost (UCS)

- Enqueue nodes by **path cost**. That is, let g(n) = cost of the path from the start node to the current node n. Sort nodes by increasing value of g.
- Called *"Dijkstra's Algorithm"* in the algorithms literature and similar to *"Branch and Bound Algorithm"* in operations research literature
- **Complete (*)**
- **Optimal/Admissible** (*)
- Admissibility depends on the goal test being applied *when a node is removed from the nodes list*, not when its parent node is expanded and the node is first generated
- **Exponential time and space complexity**, O(b^d)

Depth-First Iterative Deepening (DFID)

- First do DFS to depth 0 (i.e., treat start node as having no successors), then, if no solution found, do DFS to depth 1, etc.

```
until solution found do
  DFS with depth cutoff c
  c = c+1
```

- **Complete**
- **Optimal/Admissible** if all operators have the same cost. Otherwise, not optimal but guarantees finding solution of shortest length (like BFS).
- Time complexity seems worse than BFS or DFS because nodes near the top of the search tree are generated multiple times, but because almost all of the nodes are near the bottom of a tree, the worst case time complexity is still exponential, O(b^d).

Depth-First Iterative Deepening

- If branching factor is b and solution is at depth d, then nodes at depth d are generated once, nodes at depth d-1 are generated twice, etc.
  - IDS : $(d)\ b + (d-1)\ b^2 + \ldots + (2)\ b^{(d-1)} + b^d = O(b^d)$.
  - If b=4, then worst case is $1.78 * 4^d$, i.e., 78% more nodes searched than exist at depth d (in the worst case).
- However, let's compare this to the time spent on BFS:
  - BFS : $b + b^2 + \ldots + b^d + (b^{(d+1)} - b) = O(b^d)$.
  - Same time complexity of $O(b^d)$, but BFS expands some nodes at depth d+1, which can make a HUGE difference:
    - With b = 10, d = 5,
    - BFS: 10 + 100 + 1,000 + 10,000 + 100,000 + 999,990 = 1,111,100
    - IDS: 50 + 400 + 3,000 + 20,000 + 100,000 = 123,450
- IDS can actually be quicker in-practice than BFS, even though it regenerates early states.

Depth-First Iterative Deepening

- **Exponential time complexity,** O(b^d), like BFS
- **Linear space complexity**, O(bd), like DFS

- Has advantage of BFS (i.e., completeness) and also advantages of DFS (i.e., limited space and finds longer paths more quickly)
- Generally preferred for **large state spaces** where **solution depth is unknown**

Uninformed Search Results

Breadth-First Search

Expanded node	**Nodes list**
	{ S^0 }
S^0	{ A^3 B^1 C^8 }
A^3	{ B^1 C^8 D^6 E^{10} G^{18} }
B^1	{ C^8 D^6 E^{10} G^{18} G^{21} }
C^8	{ D^6 E^{10} G^{18} G^{21} G^{13} }
D^6	{ E^{10} G^{18} G^{21} G^{13} }
E^{10}	{ G^{18} G^{21} G^{13} }
G^{18}	{ G^{21} G^{13} }

Solution path found is S A G , cost 18

Number of nodes expanded (including goal node) = 7

Depth-First Search

Expanded node	Nodes list
	{ S^0 }
S^0	{ A^3 B^1 C^8 }
A^3	{ D^6 E^{10} G^{18} B^1 C^8 }
D^6	{ E^{10} G^{18} B^1 C^8 }
E^{10}	{ G^{18} B^1 C^8 }
G^{18}	{ B^1 C^8 }

Solution path found is S A G, cost 18

Number of nodes expanded (including goal node) = 5

Uniform-Cost Search

Expanded node	**Nodes list**
	{ S^0 }
S^0	{ B^1 A^3 C^8 }
B^1	{ A^3 C^8 G^{21} }
A^3	{ D^6 C^8 E^{10} G^{18} G^{21} }
D^6	{ C^8 E^{10} G^{18} G^1 }
C^8	{ E^{10} G^{13} G^{18} G^{21} }
E^{10}	{ G^{13} G^{18} G^{21} }
G^{13}	{ G^{18} G^{21} }

Solution path found is S B G, cost 13

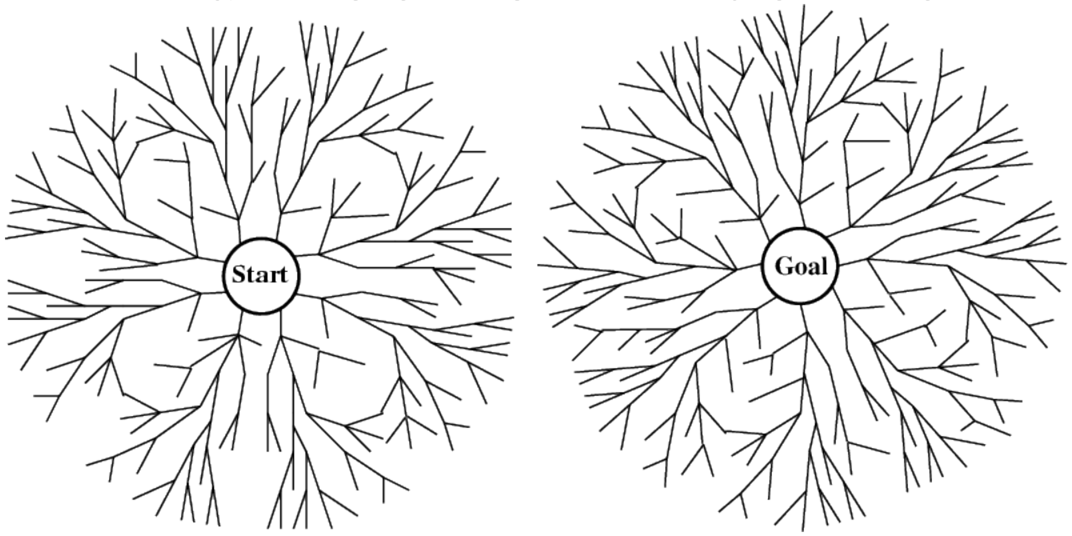
Number of nodes expanded (including goal node) = 7

How they perform

- **Breadth-First Search**:
  - Expanded nodes: S A B C D E G
  - Solution found: S A G (cost 18)
- **Depth-First Search:**
  - Expanded nodes: S A D E G
  - Solution found: S A G (cost 18)
- **Uniform-Cost Search**:
  - Expanded nodes: S A D B C E G
  - Solution found: S B G (cost 13)

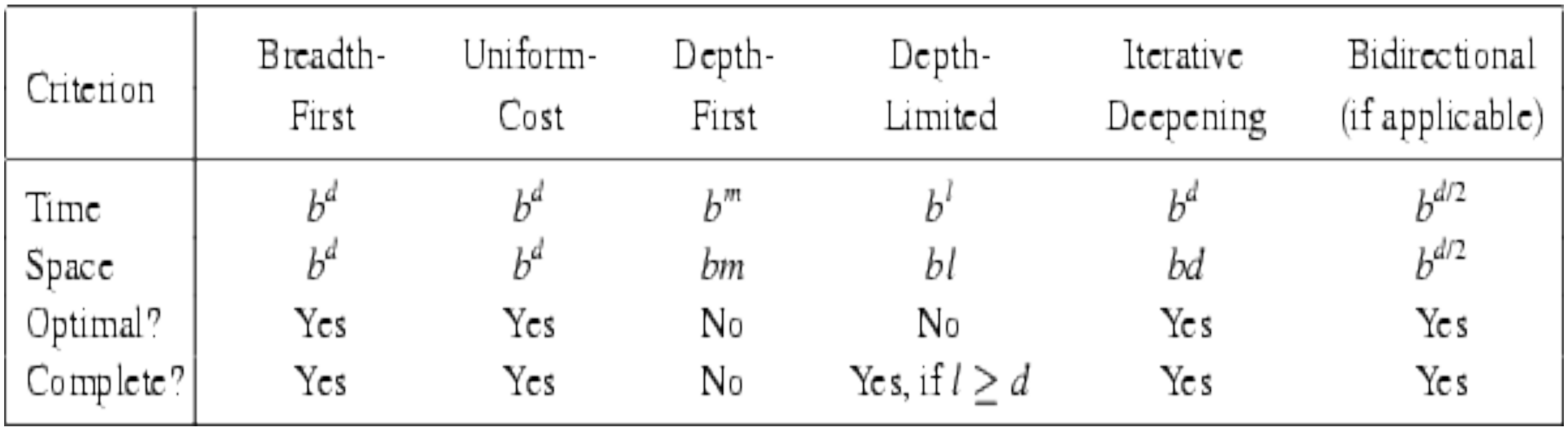
  This is the only uninformed search that worries about costs.
- **Iterative-Deepening Search**:
  - nodes expanded: S S A B C S A D E G
  - Solution found: S A G (cost 18)

Bi-directional search



- Alternate searching from the start state toward the goal and from the goal state toward the start.
- Stop when the frontiers intersect.
- Works well only when there are unique start and goal states.
- Requires the ability to generate “predecessor” states.
- Can (sometimes) lead to finding a solution more quickly.
- Time complexity: $O(b^{d/2})$. Space complexity: $O(b^{d/2})$.

Comparing Search Strategies



Criterion	Breadth-First	Uniform-Cost	Depth-First	Depth-Limited	Iterative Deepening	Bidirectional (if applicable)
Time	b^d	b^d	b^m	b^l	b^d	$b^{d/2}$
Space	b^d	b^d	bm	bl	bd	$b^{d/2}$
Optimal?	Yes	Yes	No	No	Yes	Yes
Complete?	Yes	Yes	No	Yes, if $l \geq d$	Yes	Yes

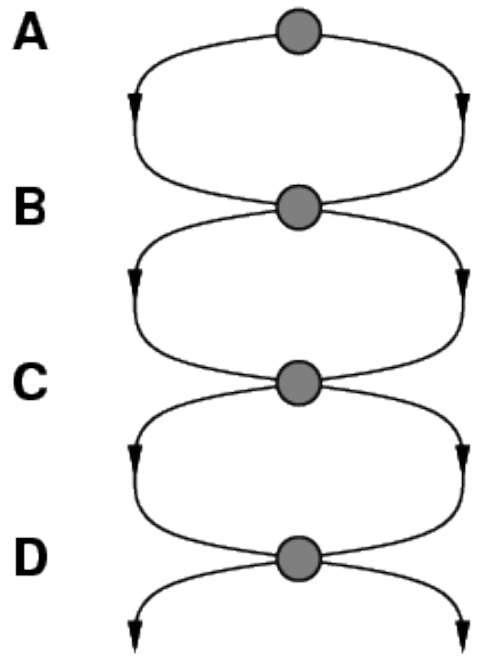
b – branching factor
m – maximum depth

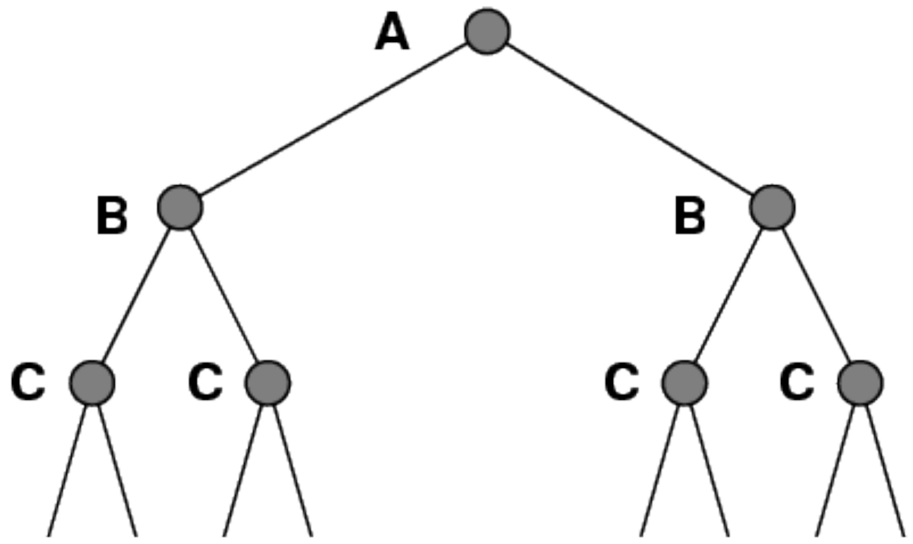
d – depth of optimal solution
l – depth limit

Avoiding Repeated States

- In increasing order of effectiveness in reducing size of state space and with increasing computational costs:
  1. Do not return to the state you just came from.
  2. Do not create paths with cycles in them.
  3. Do not generate any state that was ever created before.
- Net effect depends on frequency of “loops” in state space.

A State Space that Generates an Exponentially Growing Search Space





Graph Search

```
function graph-search (problem, QUEUEING-FUNCTION)
;; problem describes the start state, operators, goal test, and operator costs
;; queueing-function is a comparator function that ranks two states
;; graph-search returns either a goal node or failure
nodes = MAKE-QUEUE(MAKE-NODE(problem.INITIAL-STATE))
  closed = {}
   loop
      if EMPTY(nodes) then return "failure"
      node = REMOVE-FRONT(nodes)
      if problem.GOAL-TEST(node.STATE) succeeds
         then return node.SOLUTION
      if node.STATE is not in closed
         then ADD(node, closed)
              nodes = QUEUEING-FUNCTION(nodes,
                  EXPAND(node, problem.OPERATORS))
 end
   ;; Note: The goal test is NOT done when nodes are generated
   ;; Note: closed should be implemented as a hash table for efficiency
```

Graph Search Strategies

- Breadth-first search and uniform-cost search are optimal graph search strategies.
- Iterative deepening search and depth-first search can follow a non-optimal path to the goal.
- Iterative deepening search can be used with modification:
  - It must check whether a new path to a node is better than the original one
  - If so, IDS must revise the depths and path costs of the node's descendants.

Holy Grail Search

Expanded node	**Nodes list**
	$\{ S^0 \}$
S^0	$\{C^8\ A^3\ B^1 \}$
C^8	$\{ G^{13}\ A^3\ B^1 \}$
G^{13}	$\{ A^3\ B^1 \}$

Solution path found is S C G, cost 13 **(optimal)**

Number of nodes expanded (including goal node) = 3
(as few as possible!)

If only we knew where we were headed…