Reminder that project checkpoint 1 is also due next week, on Friday 10/6 at 12pm. This homework is therefore one question shorter.

1. 4.9

2. Full write-up for (c). You are working in the warehouse of an online store and are given the task to decide how to best stack a set of \( n \) boxes. Each of the \( b_i \) (\( 1 \leq i \leq n \)) box has an associated weight \( w_i \). Customers may purchase merchandises contained in any of the boxes at any time, which will require you to retrieve the corresponding box. You are told that the probability of sale for each box \( b_i \) is \( p_i \), where \( 0 \leq p_i \leq 1 \), and \( \sum_{i=1}^{n} p_i = 1 \). In addition, since the boxes are stacked on top of each other, all boxes on top of the desired one must be lifted off before it can be accessed.

A stacking of boxes can be specified by a permutation \( S = \langle s_1, \ldots, s_n \rangle \) of the numbers \( \{1, \ldots, n\} \). Given a stacking \( S \), the individual cost of getting to the \( i \)th box is the product of its sales probability and the sum of weight of all boxes above it and itself, that is, \( C_i(S) = p_{s_i} \cdot (\sum_{j=1}^{i} w_{s_j}) \). The total cost of a stacking \( S \) is the sum of all individual costs, \( T(S) = \sum_{i=1}^{n} C_i(S) \).

For example, given the following 4 boxes:

\[
\begin{align*}
b_1 : & \quad w_1 = 300 \quad p_1 = 0.4 \\
b_2 : & \quad w_2 = 200 \quad p_2 = 0.35 \\
b_3 : & \quad w_3 = 500 \quad p_3 = 0.1 \\
b_4 : & \quad w_4 = 100 \quad p_4 = 0.15
\end{align*}
\]

The following stacking of \( S = \langle s_1 = 4, s_2 = 2, s_3 = 1, s_4 = 3 \rangle \) results in a total cost of

\[
T(S) = 0.15 \cdot 100 + 0.35 \cdot (100 + 200) + 0.4 \cdot (100 + 200 + 300) + 0.1 \cdot (100 + 200 + 300 + 500) = 470
\]

You may disregard any concerns of ceiling height, as well as dimensions and mechanical strengths of the boxes. In other words, assume that all permutations give stacking orders that are physically reasonable.

(a) Present a (short) counterexample to show that stacking the boxes in increasing order of weight \( w_i \) is not optimal

(b) Present a (short) counterexample to show that stacking the boxes in decreasing order of access probability \( p_i \) is not optimal

(c) Present an algorithm, which given a list of \( w_i \) and \( p_i \), determines a stacking \( S \) of minimum total cost. Prove your algorithm’s correctness and derive its running time.
3. Full write-up. Suppose you are standing in the open surrounded by enemies, represented by circles of different sizes. You want to use your laser gun to destroy the enemies without moving from your current location. The enemies are stationary, your laser beam penetrates and you want to fire as few shots as possible.

The problem can be stated more formally as follows. Given a set $C$ of $n$ circles in the plane, each specified by its radius and the $(x, y)$ coordinates of its center, compute the minimum number of rays from the origin that intersect every circle in $C$. Your goal is to find an efficient algorithm for this problem. This is a much simplified version of the more general “shooter-location problem”, which has applications in graphics, robotics and image processing.

Assume that you have a function `intersect(r, c)` which determines whether an arbitrary ray $r$ intersects an arbitrary circle $c$ in $O(1)$ time. This is indeed true and easy enough to write, but not the interesting part of this problem. Assume also that it is possible to shoot a ray that does not intersect any circles.

Please hand in your assignment in class.