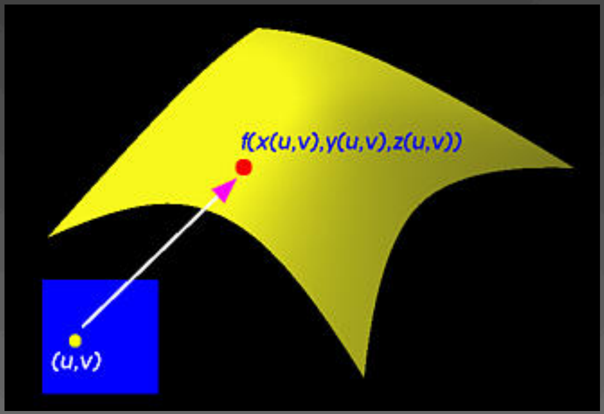
Computer Graphics

Surfaces

Based on slides by Dianna Xu, Bryn Mawr College

Parametric Surfaces

- Generalizing from curves to surfaces by using two parameters **`u`** and **`v`**



- Parametric surfaces can be either rectangular or triangular, depending on how the parameter plane is divided

Parametric Surfaces

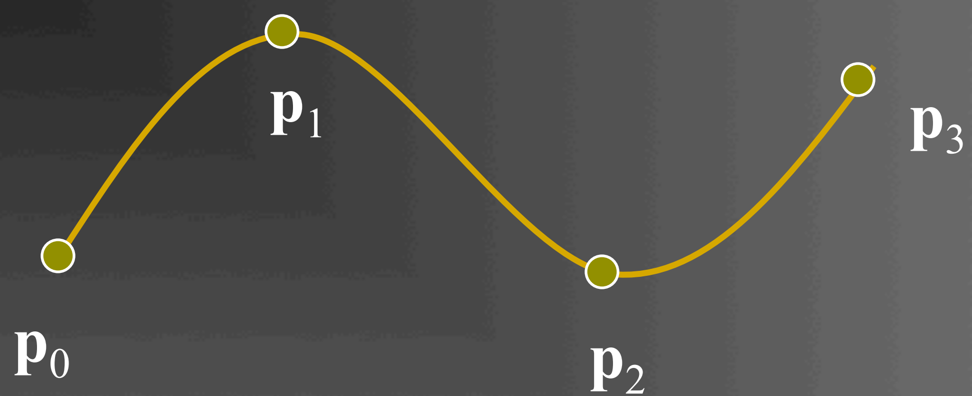
- Parametric surface:

$$p(u,v) = \begin{bmatrix} f_x(u,v) \\ f_y(u,v) \\ f_z(u,v) \end{bmatrix} = \sum_{i=0}^{n} \sum_{j=0}^{m} C_{i,j} u^i v^j$$

- Cubic interpolating patch:

$$p(u,v) = \sum_{i=0}^{3} \sum_{j=0}^{3} b_i(u) b_j(v) p_{ij}$$

Interpolating Curve



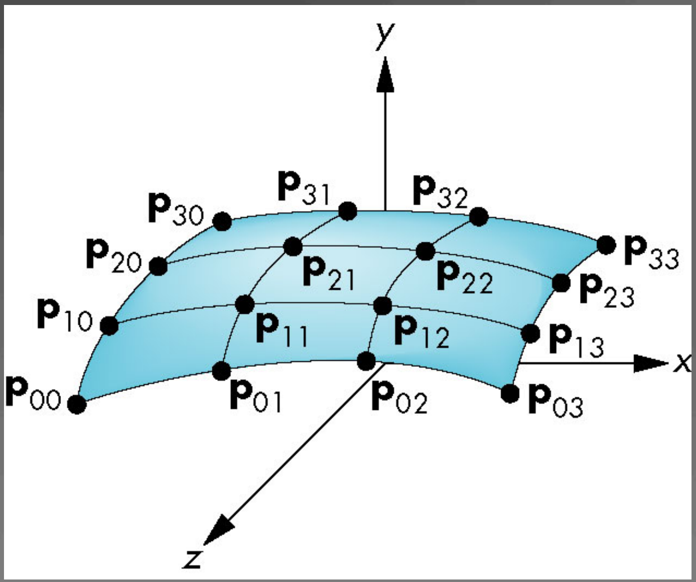
Given four data (control) points $\mathbf{p}_0$, $\mathbf{p}_1$,$\mathbf{p}_2$, $\mathbf{p}_3$ determine cubic $\mathbf{p}(u)$ which passes through them

Must find $\mathbf{c}_0$,$\mathbf{c}_1$,$\mathbf{c}_2$, $\mathbf{c}_3$

Interpolating Patch

Need 16 conditions to determine the 16 coefficients c_{ij}

Choose at $u,v = 0, 1/3, 2/3, 1$



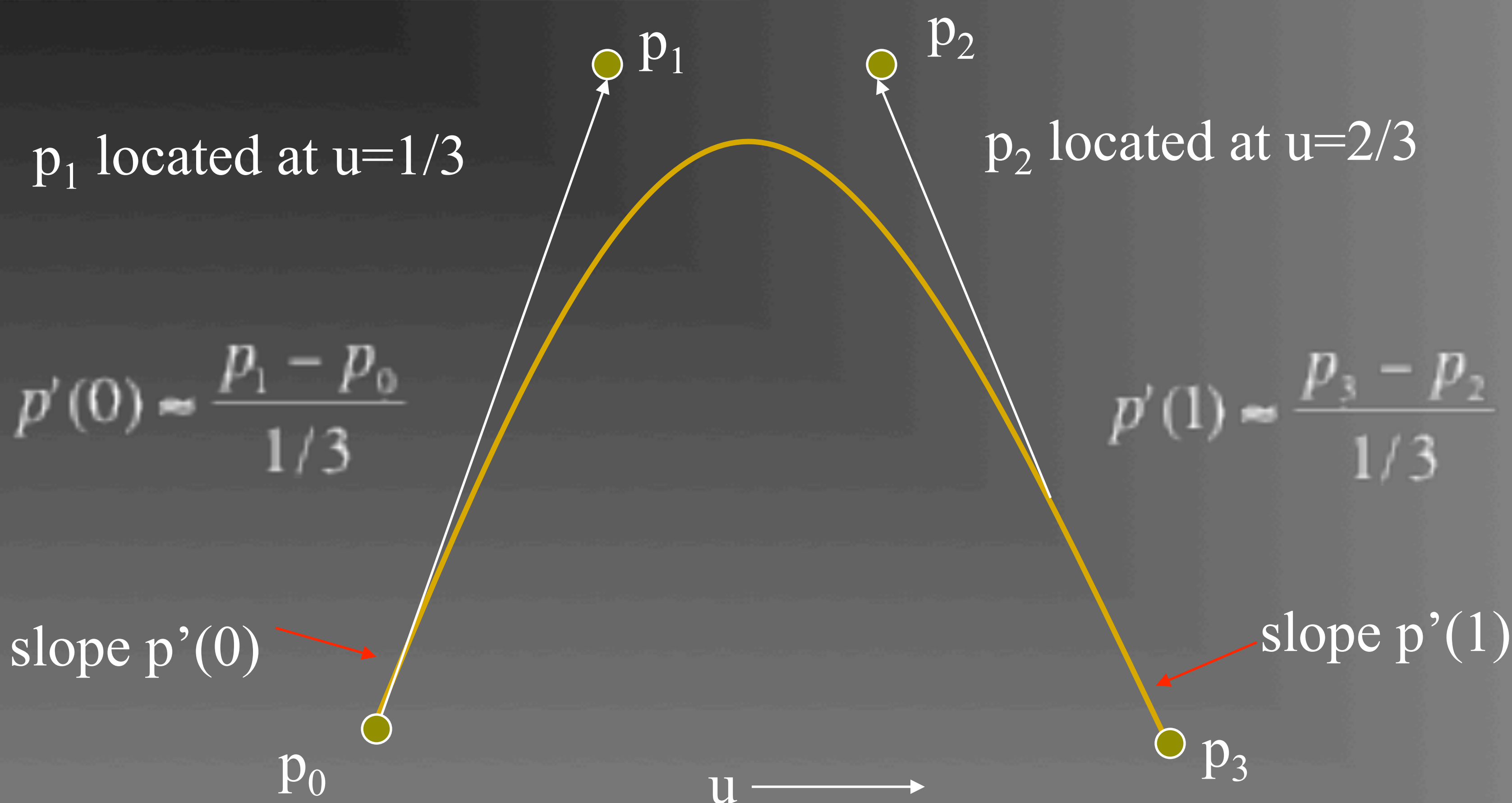
Approximating Derivatives

p_1 located at $u=1/3$

p_2 located at $u=2/3$

$$p'(0) \approx \frac{p_1 - p_0}{1/3}$$

$$p'(1) \approx \frac{p_3 - p_2}{1/3}$$



Bezier Matrix

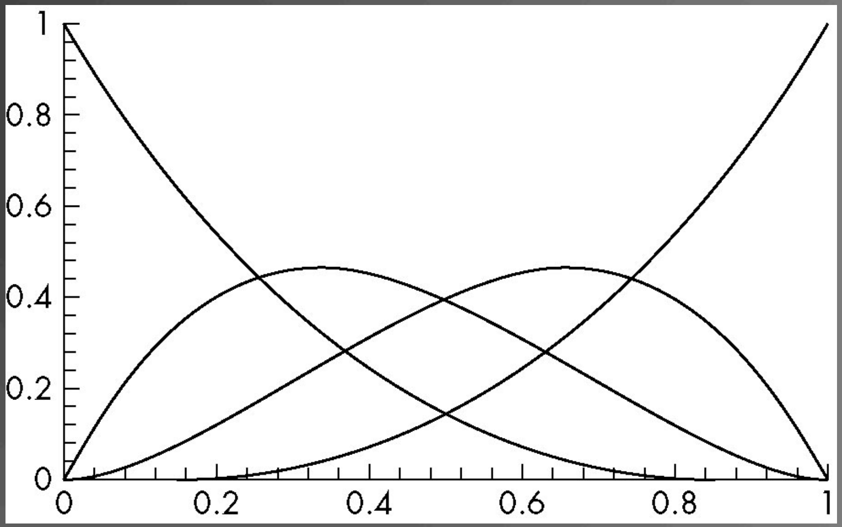
$$M_B = \begin{bmatrix} 1 & 0 & 0 & 0 \\ -3 & 3 & 0 & 0 \\ 3 & -6 & 3 & 0 \\ -1 & 3 & -3 & 1 \end{bmatrix}$$

$$p(u) = u^T M_B P = b(u)^T P$$

blending functions

Blending Functions

$$b(u) = \begin{bmatrix} (1-u)^3 \\ 3u(1-u)^2 \\ 2u^2(1-u) \\ u^3 \end{bmatrix}$$



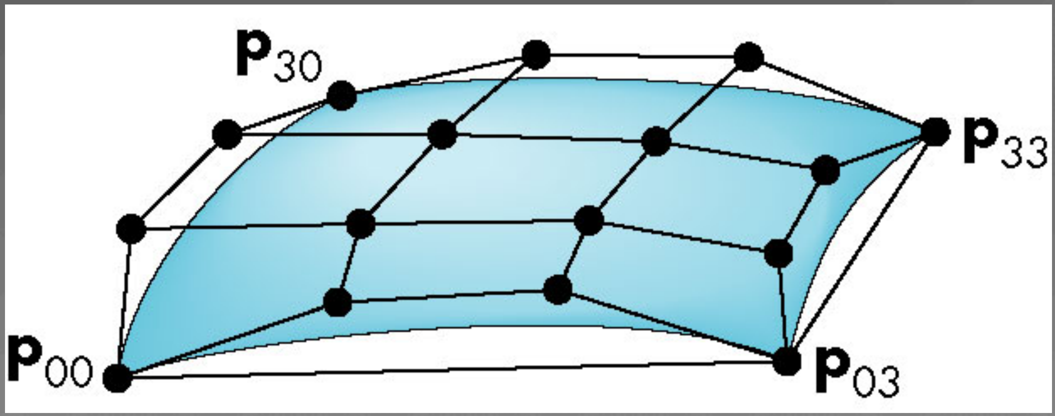
Note that all zeros are at 0 and 1 which forces the functions to be smooth over (0,1)

Bezier Patches

Using same data array $\mathbf{P}=[p_{ij}]$ as with interpolating form

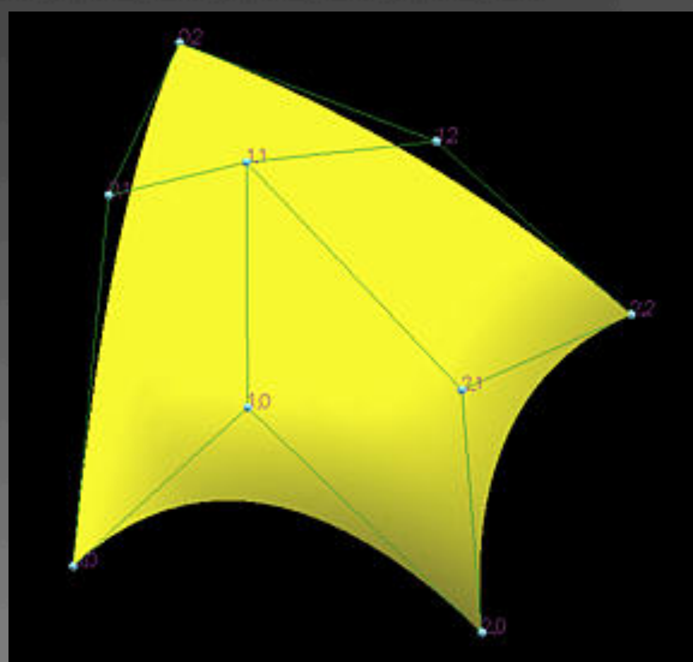
$$p(u,v)=\sum_{i=0}^{3}\sum_{j=0}^{3} b_i(u)b_j(v)p_{ij}=u^T M_B P M_B^T v$$

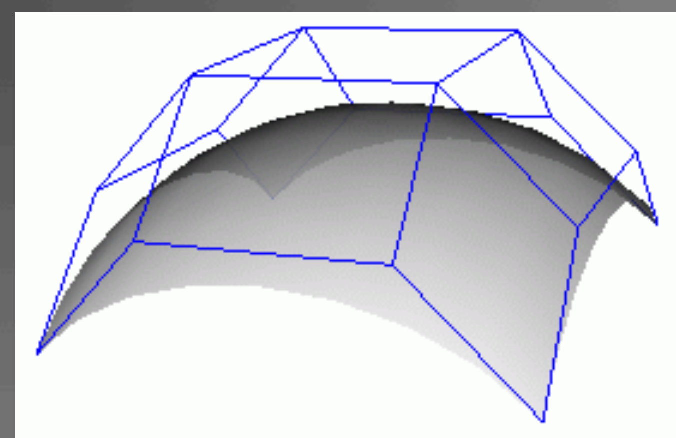
Patch lies in convex hull



Bézier Surfaces

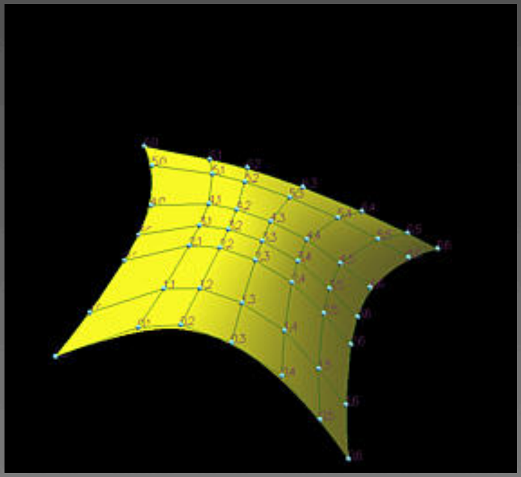
- Defined in terms of a two dimensional control net

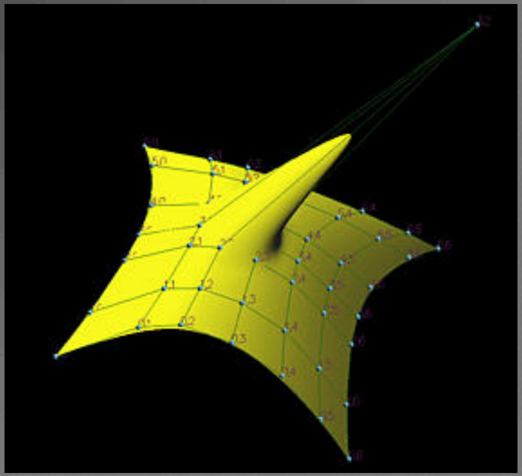




B-spline Surfaces: local flexibility

- Local flexibility is one of the most desirable properties of B-splines
- Modification of a control point only affects a small neighborhood

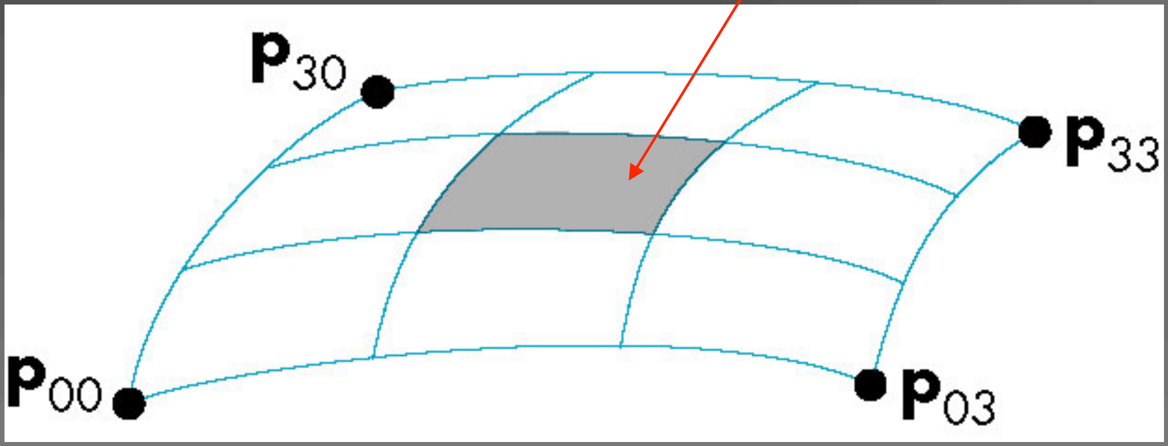




B-Spline Patches

$$p(u,v) = \sum_{i=0}^{3}\sum_{j=0}^{3} b_i(u) b_j(v) p_{ij} = u^T M_S P M_S^T v$$

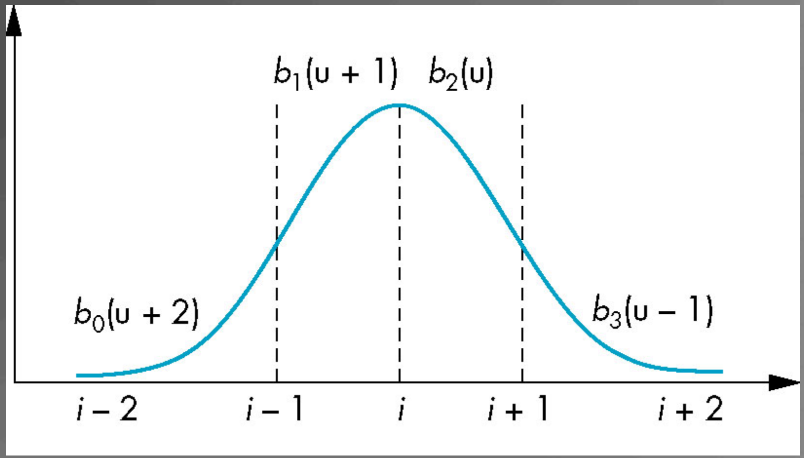
defined over only 1/9 of region



Basis Functions

In terms of the blending polynomials

$$B_i(u) = \begin{cases} 0 & u < i-2 \\ b_0(u+2) & i-2 \le u \le i-1 \\ b_1(u+1) & i-1 \le u \le i \\ b_2(u) & i \le u \le i+1 \\ b_3(u-1) & i+1 \le u \le i+2 \\ 0 & u \ge i+2 \end{cases}$$



Evaluating Polynomials

- Simplest method to render a polynomial curve is to evaluate the polynomial at many points and form an approximating polyline
- For surfaces we can form an approximating mesh of triangles or quadrilaterals
- Use Horner's method to evaluate polynomials

$$p(u) = c_0 + u(c_1 + u(c_2 + uc_3))$$

  - 3 multiplications/evaluation for cubic

Finite Differences

For equally spaced $\{u_k\}$ we define *finite differences*

$$\Delta^{(0)} p(u_k) = p(u_k)$$

$$\Delta^{(1)} p(u_k) = p(u_{k+1}) - p(u_k)$$

$$\Delta^{(m+1)} p(u_k) = \Delta^{(m)} p(u_{k+1}) - \Delta^{(m)} p(u_k)$$

For a polynomial of degree n, the n^{th} finite difference is constant

Building a Finite Difference Table

$$p(u) = 1 + 3u + 2u^2 + u^3$$

t	0	1	2	3	4	5
$\mathbf{p}$	1	7	23	55	109	191
$\Delta^{(1)}\mathbf{p}$	6	16	32	54	82	
$\Delta^{(2)}\mathbf{p}$	10	16	22	28		
$\Delta^{(3)}\mathbf{p}$	6	6	6			

Finding the Next Values

Starting at the bottom, we can work up generating new values for the polynomial

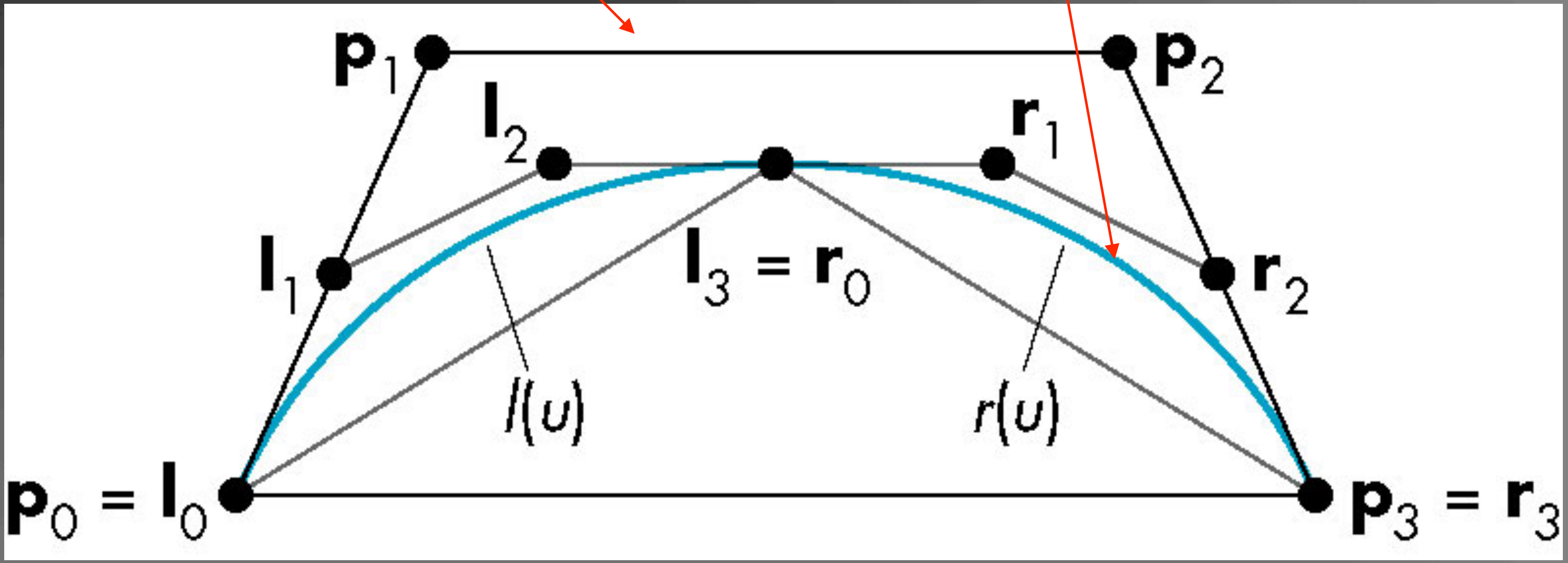
t	0	1	2	3	4	5
$\mathbf{p}$	1	7	23	55	109	191
$\Delta^{(1)}\mathbf{p}$	6	16	32	54	82	
$\Delta^{(2)}\mathbf{p}$	10	16	22	28		
$\Delta^{(3)}\mathbf{p}$	6	6	6			

de Casteljau Recursion

- We can use the convex hull property of Bezier curves to obtain an efficient recursive method that does not require any function evaluations
- Uses only the values at the control points
- Repeatedly refine the control polygon until point on curve is reached.

Splitting a Cubic Bezier

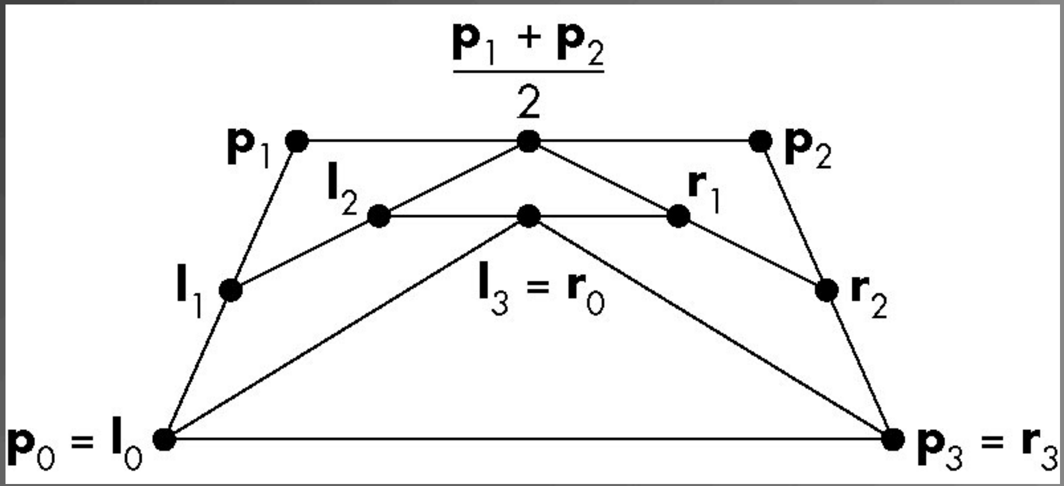
p_0, p_1, p_2, p_3 determine a cubic Bezier polynomial and its convex hull



Consider left half $l(u)$ and right half $r(u)$

Efficient Form

$l_0 = p_0$
$r_3 = p_3$
$l_1 = ½(p_0 + p_1)$
$r_2 = ½(p_2 + p_3)$
$l_2 = ½(l_1 + ½(p_1 + p_2))$
$r_1 = ½(r_2 + ½(p_1 + p_2))$
$l_3 = r_0 = ½(l_2 + r_1)$



Requires only shifts and adds!

Every Curve is a Bezier Curve

- We can render a given polynomial using the recursive method if we find control points for its representation as a Bezier curve.
- Suppose that p(u) is given as an interpolating curve with control points Q.

$$p(u) = u^T M_I Q$$

- There exist Bezier control points P such that

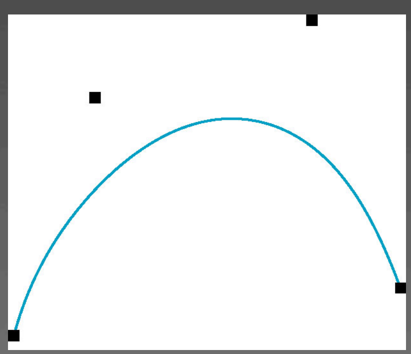
$$p(u) = u^T M_B P$$

- Equating and solving, we find

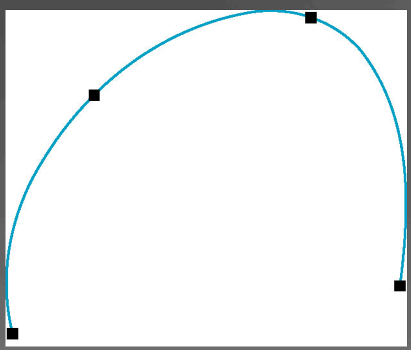
$$P = M_B^{-1} M_I Q$$

Example

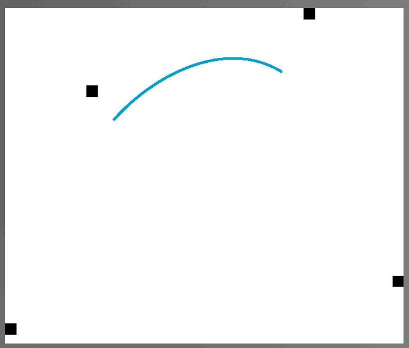
These three curves were all generated from the same original data using Bezier recursion by converting all control point data to Bezier control points



Bezier



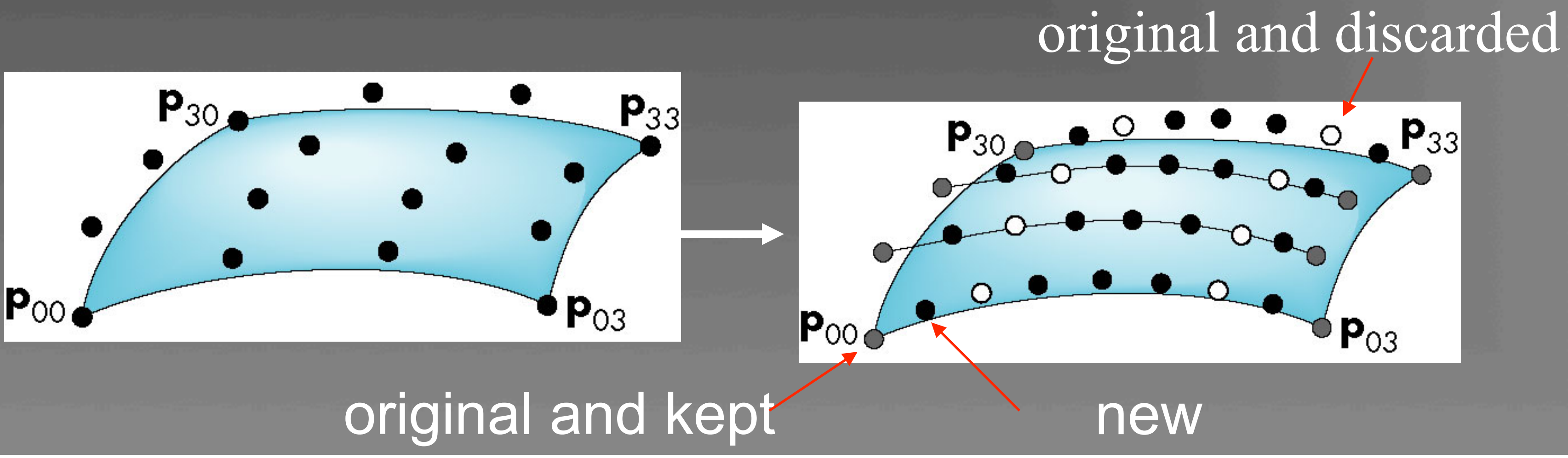
Interpolating



B Spline

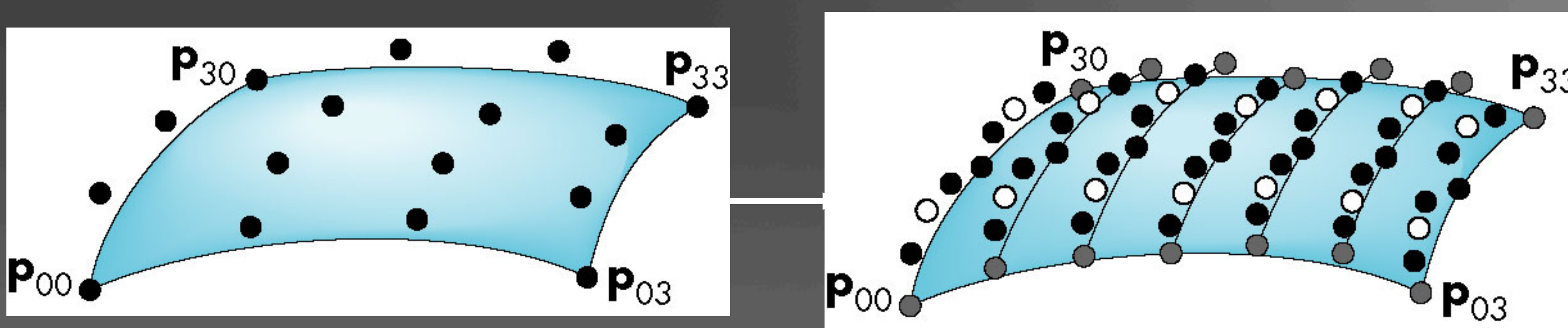
Surfaces

- Can apply the recursive method to surfaces if we recall that for a Bezier patch curves of constant u (or v) are Bezier curves in u (or v)
- First subdivide in u
  - Process creates new points
  - Some of the original points are discarded

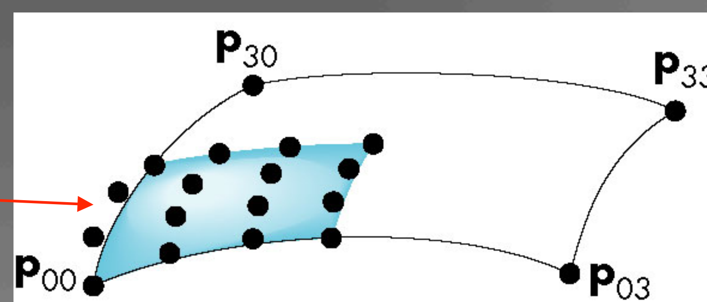


Second Subdivision

- ● New points created by subdivision
- ○ Old points discarded after subdivision
- ◍ Old points retained after subdivision

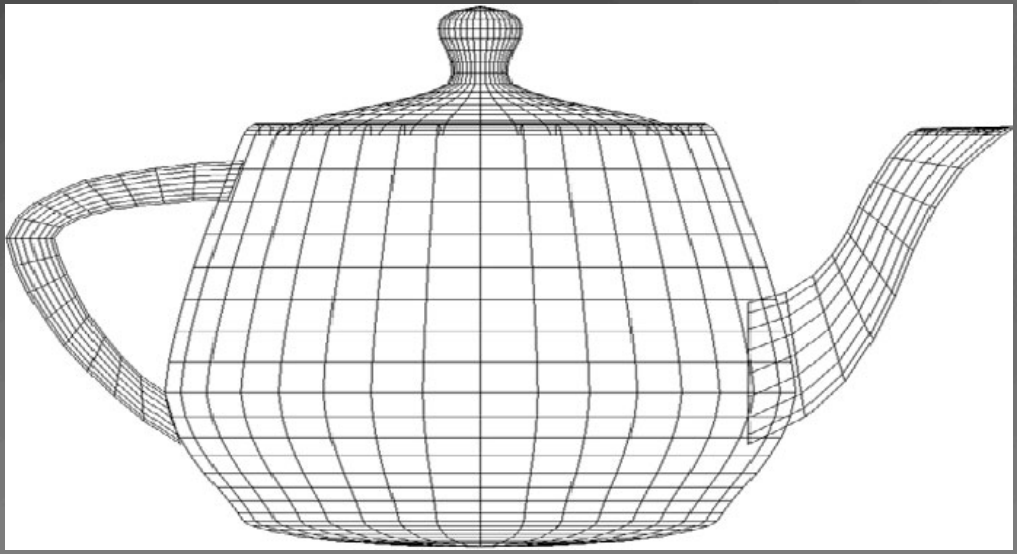


16 final points for
1 of 4 patches created



Utah Teapot

- Most famous data set in computer graphics
- Widely available as a list of 306 3D vertices and the indices that define 32 Bezier patches



What Does OpenGL Support?

- Evaluators: a general mechanism for working with the Bernstein polynomials
  - Can use any degree polynomials
  - Can use in 1-4 dimensions
  - Automatic generation of normals and texture coordinates
  - NURBS supported in GLU
- Quadrics
  - GLU and GLUT contain polynomial approximations of quadrics

One-Dimensional Evaluators

- Evaluate a Bernstein polynomial of any degree at a set of specified values
- Can evaluate a variety of variables
  - Points along a 2, 3 or 4 dimensional curve
  - Colors
  - Normals
  - Texture Coordinates
- We can set up multiple evaluators that are all evaluated for the same value

Setting Up an Evaluator

what we want to evaluate

max and min of u

```
glMap1f(type,u_min,u_max,stride,
        order, pointer_to_array)
```

separation between data points

1+degree of polynomial

pointer to control data

Each type must be enabled by `glEnable(type)`

Example

Consider an evaluator for a cubic Bezier curve over (0,1)

```
Point cpoints[]={...........}; * /3d data /*
glMap1f(GL_MAP_VERTEX_3,0.0,1.0,3,4,cpoints);
```

data are 3D vertices

cubic

data are arranged as x,y,z,x,y,z……
three floats between data points in array

```
glEnable(GL_MAP_VERTEX_3);
```

Evaluating

- The function `glEvalCoord1f(u)` causes all enabled evaluators to be evaluated for the specified **u**
  - Can replace `glVertex`, `glNormal`, `glTexCoord`
- The values of **u** need not be equally spaced

Example

- Consider the previous evaluator that was set up for a cubic Bezier over (0,1)
- Suppose that we want to approximate the curve with a 100 point polyline

```
glBegin(GL_LINE_STRIP)
  for(i=0; i<100; i++)
    glEvalCoord1f( (float) i/100.0);
glEnd();
```

Equally Spaced Points

Rather than using a loop, we can set up an equally spaced mesh (grid) and then evaluate it with one function call

```
glMapGrid(100, 0.0, 1.0);
```

sets up 100 equally-spaced points on (0,1)

```
glEvalMesh1(GL_LINE, 0, 99);
```

renders lines between adjacent evaluated points from point 0 to point 99

Bezier Surfaces

- Similar procedure to 1D but use 2D evaluators in **`u`** and **`v`**

```
glMap2f(type, u_min, umax, u_stride,
u_order, v_min, v_max, v_stride,
v_order, pointer_to_data)
```

- Evaluate with **`glEvalCoord2f(u,v)`**

Example

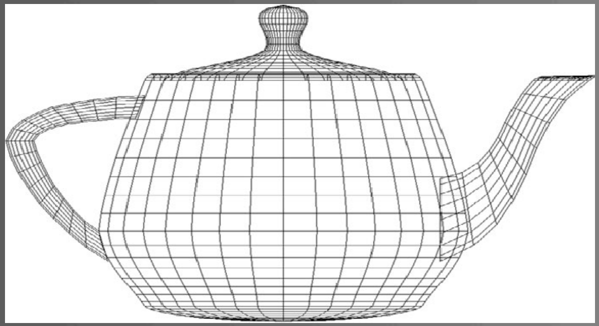
bicubic over (0,1) x (0,1)

```
Point cpoints[4][4]={.........};
glMap2f(GL_MAP_VERTEX_3, 0.0, 1.0, 3, 4,
    0.0, 1.0, 12, 4, cpoints);
```

Note that in v direction data points are separated by 12 floats since array **`data`** is stored by rows

Rendering with Lines

must draw in both directions



```
for(j=0;j<100;j++) {
  glBegin(GL_LINE_STRIP);
    for(i=0;i<100;i++)
       glEvalCoord2f((float) i/100.0, (float) j/100.0);
  glEnd();
  glBegin(GL_LINE_STRIP);
    for(i=0;i<100;i++)
       glEvalCoord2f((float) j/100.0, (float) i/100.0);
  glEnd();
}
```

Rendering with Quadrilaterals

Form a quad mesh and render with lines

```
for(j=0; j<99; j++) {
  glBegin(GL_QUAD_STRIP);
    for(i=0; i<100; i++) {
      glEvalCoord2f ((float) i/100.0,
          (float) j/100.0);
      glEvalCoord2f ((float)(i+1)/100.0,
          (float)j/100.0);
   }
   glEnd():
}
```

Uniform Meshes

- We can form a 2D mesh (grid) in a similar manner to 1D for uniform spacing

```
glMapGrid2(u_num, u_min, u_max,
  v_num, v_min, v_max)
```

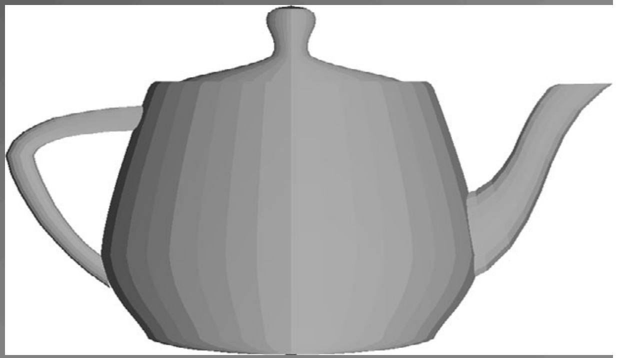
- Can evaluate as before with lines or if want filled polygons

```
glEvalMesh2( GL_FILL, u_start,
  u_num, v_start, v_num)
```

Rendering with Lighting

- If we use filled polygons, we have to shade or we will see solid color uniform rendering
- Can specify lights and materials but we need normals
  - Let OpenGL find them

```
glEnable(GL_AUTO_NORMAL);
```



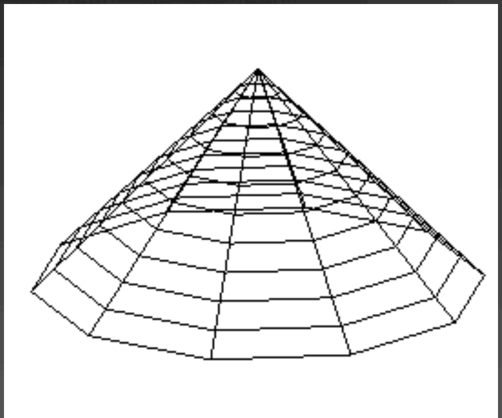
NURBS

- OpenGL supports NURBS surfaces through the GLU library
- Why GLU?
  - Can use evaluators in 4D with standard OpenGL library
  - Many complexities with NURBS that need a lot of code
  - There are five NURBS surface functions plus functions for trimming curves that can remove pieces of a NURBS surface

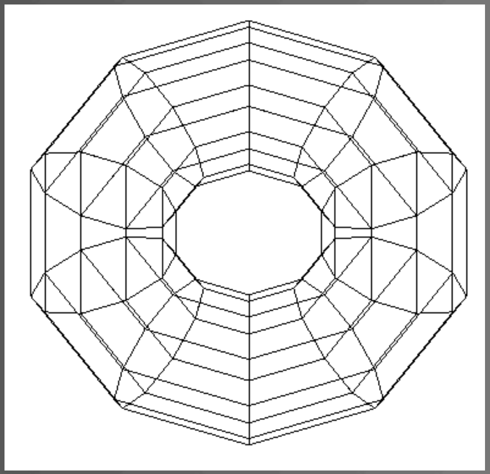
Quadrics

- Quadrics are in both the GLU and GLUT libraries
  - Both use polygonal approximations where the application specifies the resolution
  - Sphere: lines of longitude and lattitude
- GLU: disks, cylinders, spheres
  - Can apply transformations to scale, orient, and position
- GLUT: Platonic solids, torus, Utah teapot, cone

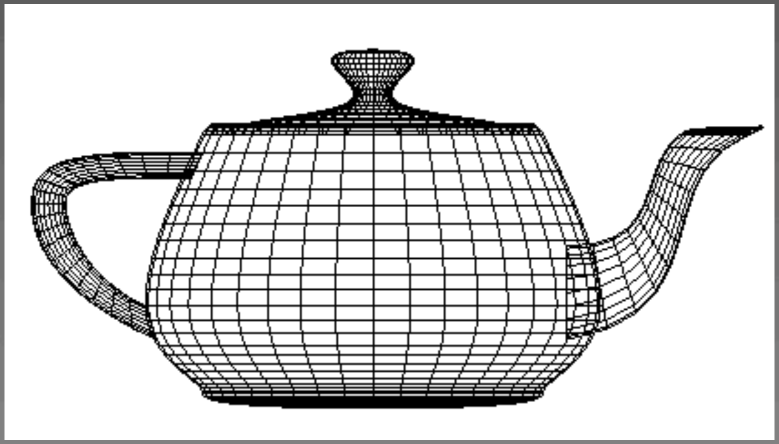
GLUT Objects



`glutWireCone()`

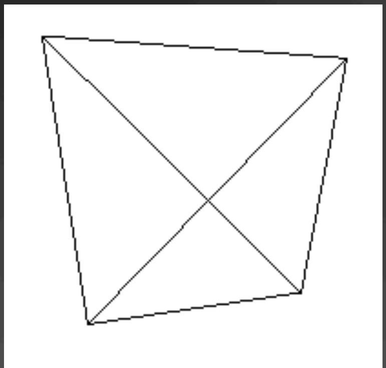


`glutWireTorus()`

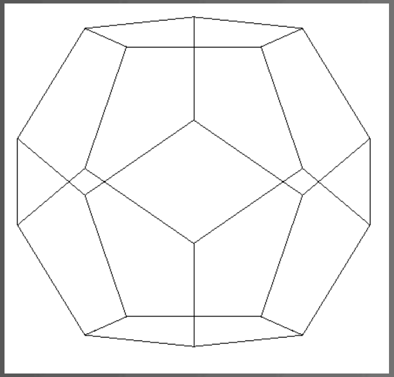


`glutWireTeapot()`

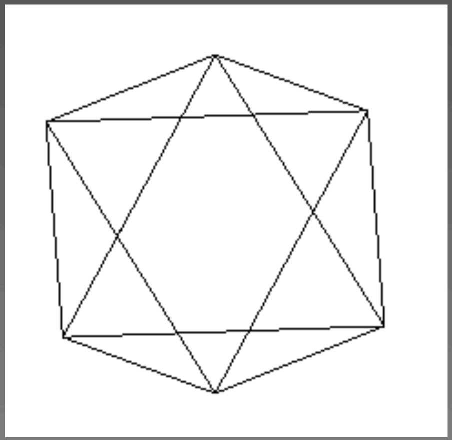
GLUT Platonic Solids



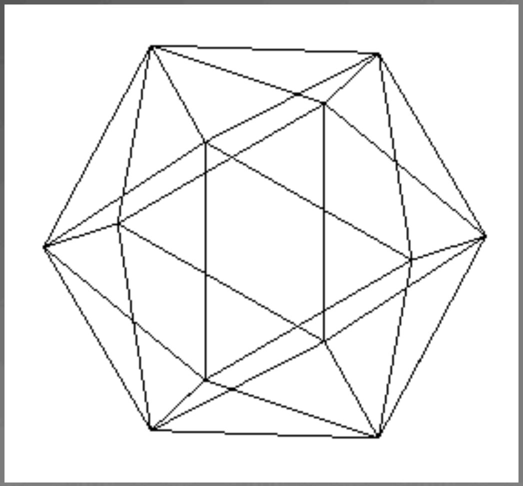
glutWireTetrahedron()



glutWireDodecahedron()



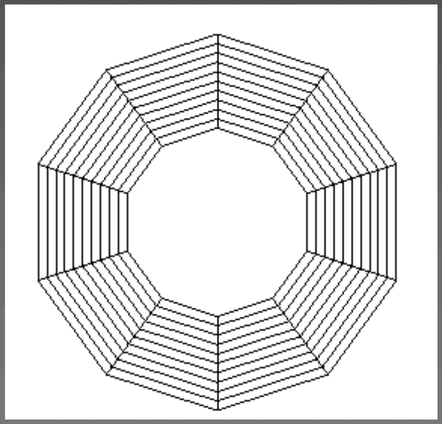
glutWireOctahedron()



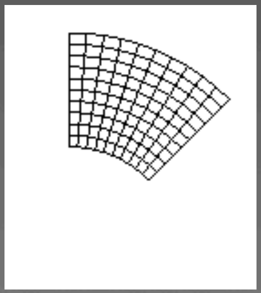
glutWireIcosahedron()

Quadric Objects in GLU

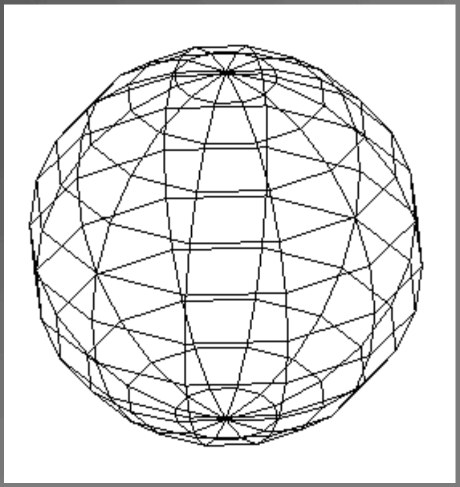
- GLU can automatically generate normals and texture coordinates
- Quadrics are objects that include properties such as how we would like the object to be rendered



disk



partial disk



sphere

Defining a Cylinder

```
GLUquadricOBJ *p;
P = gluNewQuadric(); /*set up object */
gluQuadricDrawStyle(GLU_LINE);/*render
 style*/
gluCylinder(p, BASE_RADIUS, TOP_RADIUS,
         BASE_HEIGHT, sections, slices);
```