

- Closed book, closed notes.
- You are allowed one cheat sheet (front and back) with content of your choice.
- **No** calculators or other electronic devices are allowed.
- Use the back of the exam pages if you need additional room.
- Please show all of your work. Partial credit will be given if answer is not correct but work progress shows correct intermediate steps.

1. Perform the following conversions, show conversion steps.

(a) Convert the following binary number to hexadecimal (base-16): 1101001110011010_2

(b) Convert the following base-5 number to decimal: 1234_5

(c) Find the decimal equivalent of the following 16-bit 2's complement number: 1001110000000101_2

2. Suppose that b is an integer. If $b \bmod 15 = 3$, what is $10b \bmod 15$? Why? You **MUST** justify your answer for full points.

3. Use induction to prove that for every natural number n , $3 \mid (n^3 - n)$.

4. Prove that if m is an even integer, then $m + 7$ is odd. Do this proof **in three ways**: direct proof, proof by contraposition and proof by contradiction.

5. Prove that the square root of 6 is irrational.

6. For the expression $3 + 2 + 4 + 6 + 8 + \cdots + 2n$ where $n \geq 1$, find the closed form formula and simplify it.

7. For each of the following, give a *recursive* definition. Remember to indicate the initial terms or base cases:

(a) $a_n = \sum_{i=0}^n i$

(b) The sequence that generates the terms 3, 6, 12, 24, 48, 96, 192, ...

(c) The set of non-negative even numbers

(d) The set of all even numbers

8. Let P be a polygon in the plane. To *triangulate* a polygon is to draw diagonals through the interior of the polygon so that

(a) the diagonals do not cross each other and

(b) every region created is a triangle.

Prove that a simple polygon with n sides, where n is an integer with $n \geq 3$, can be triangulated into $n - 2$ triangles.