Algorithms: Linear and Binary Search

CS 110

Bryn Mawr College

Algorithm

- A well-defined set of instructions for solving a particular kind of problem.

- Algorithms exist for systematically solving many types of problems
  - Sorting
  - Searching
  - …

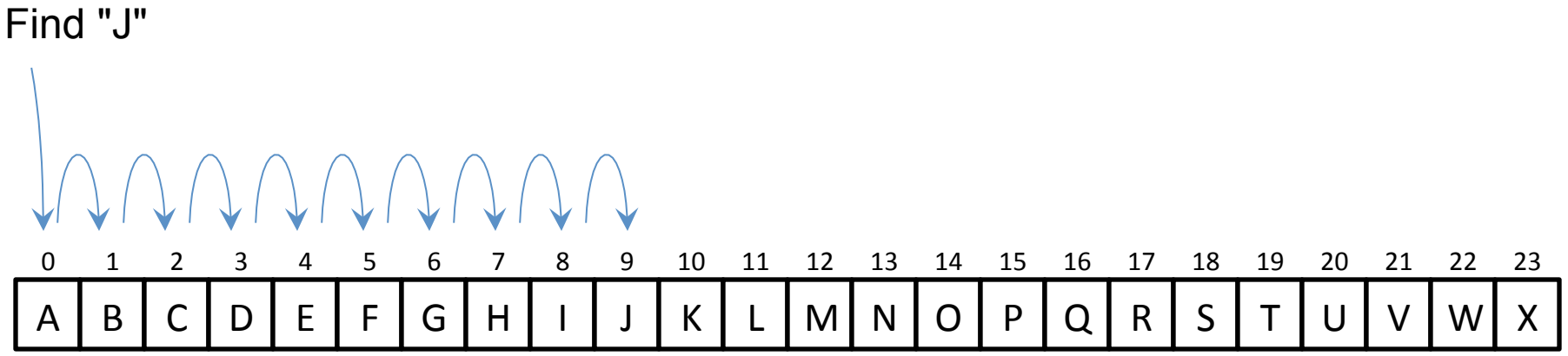
Euclid's algorithm for greatest common divisor

- Problem:
  - Find the greatest common divisor of two numbers A and B
- GCD Algorithm
  1. While B is not zero, repeat the following:
     - If A > B, then A=A-B
     - Otherwise, B=B-A
  2. A is the GCD

```
int A = 40902;
int B = 24140;

print("GCD of " + A + " and " + B + " is ");

while (B != 0) {
  if (A > B) {
    A = A - B;
  } else {
    B = B - A;
  }
}

println(A);
```

Exhaustive (Linear) Search

- Systematically enumerate all possible values and compare to value being sought
- For an array, iterate from the beginning to the end, and test each item in the array



Exhaustive (Linear) Search

```
// Search for a matching String val in the array vals.
// If found, return index. If not found, return -1.

int eSearch(String val, String[] vals) {

  // Loop over all items in the array

  for (int i=0; i<vals.length; i++) {

    // Compare items
    int rslt = val.compareTo( vals[i] );

    if ( rslt == 0 ) {              // Found it
      return i;                     // Return index
    }
  }

  return -1;      // If we get this far, val was not found.
}
```

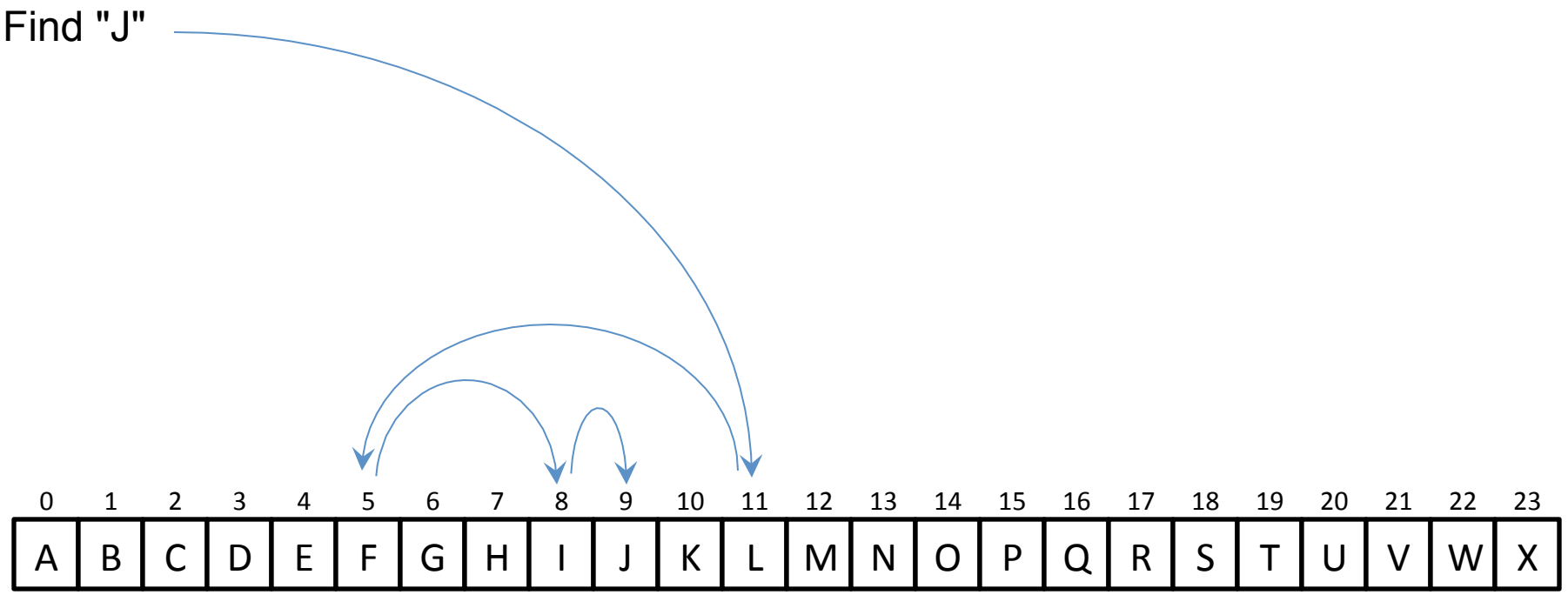
Binary Search

- Quickly find an item (val) in a sorted list.

- Procedure:
    1. Init **min** and **max** variables to lowest and highest index
    2. Repeat while **min** ≤ **max**
        a. Compare item at the **middle** index with that being sought (**val**)
        b. If **item** at **middle** equals **val**, return **middle**
        c. If **val** comes before **middle,** then reset **max** to **middle-1**
        d. If **val** comes after **middle**, reset **min** to **middle+1**
    3. If **min** > **max**, **val** not found

The most efficient way to play "guess the number" …

Binary Search

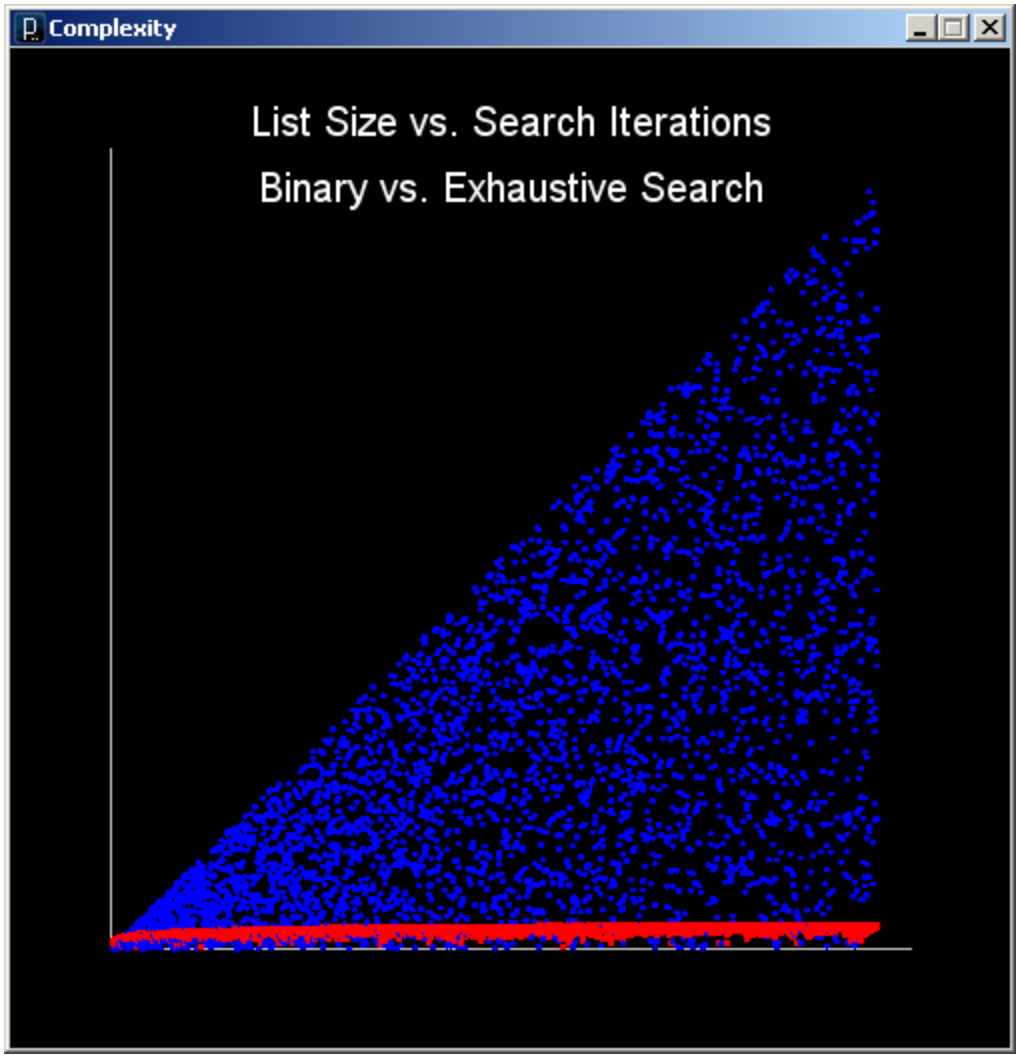


```
// Search for a matching val String in the String array vals
// If found, return index. If not found, return -1
// Use binary search.

int bSearch(String val, String[] vals) {
  int min = 0;
  int max = vals.length-1;
  int mid;
  int rslt;

  while (min <= max) {
    mid = int( (max + min)/2 );          // Compute next index

    rslt = val.compareTo( vals[mid] );  // Compare values

    if ( rslt == 0 ) {                   // Found it
      return mid;                        // Return index
    } else if ( rslt < 0 ) {             // val is before vals[mid]
      max = mid - 1;                     // Reset max to item before mid
    } else {                             // val is after vals[mid]
      min = mid + 1;                     // Reset min to item after mid
    }
  }

  // If we get this far, val was not found.
  return -1;
}
```

An Experiment - Exhaustive vs. Binary Search

- For names (Strings) in arrays of increasing size…
  - Select 10 names at random from the list
  - Search for each name using Binary and Exhaustive Search
  - Count the number of iterations it takes to find each name
  - Plot number of iterations for each against list size

- Start with an array of 3830+ names (Strings)



Wow! That's fast!

Worst Case Running Time

- Exhaustive Search

  N items in a list

  Worst case: Number of iterations = N

  (we must look at every item)

- Binary Search

  After 1^{st} iteration, N/2 items remain ($N/2^1$)

  After 2^{nd} iteration, N/4 items remain ($N/2^2$)

  After 3^{rd} iteration, N/8 items remain ($N/2^3$)

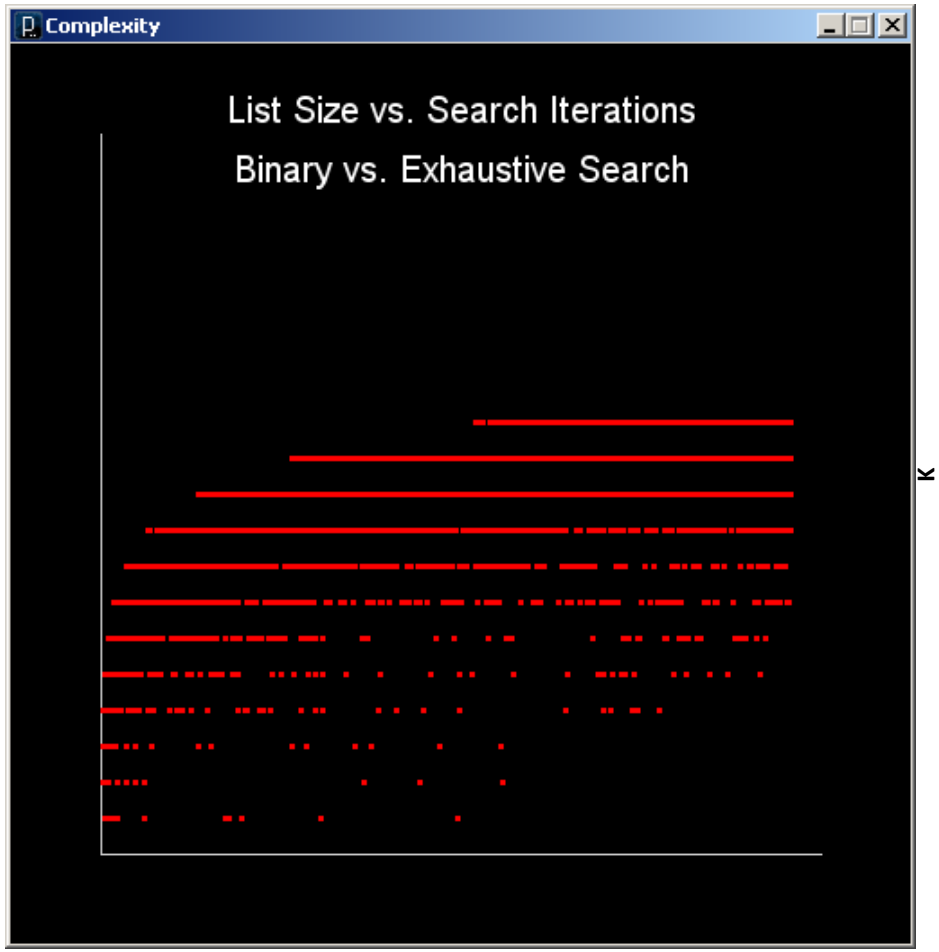
  …

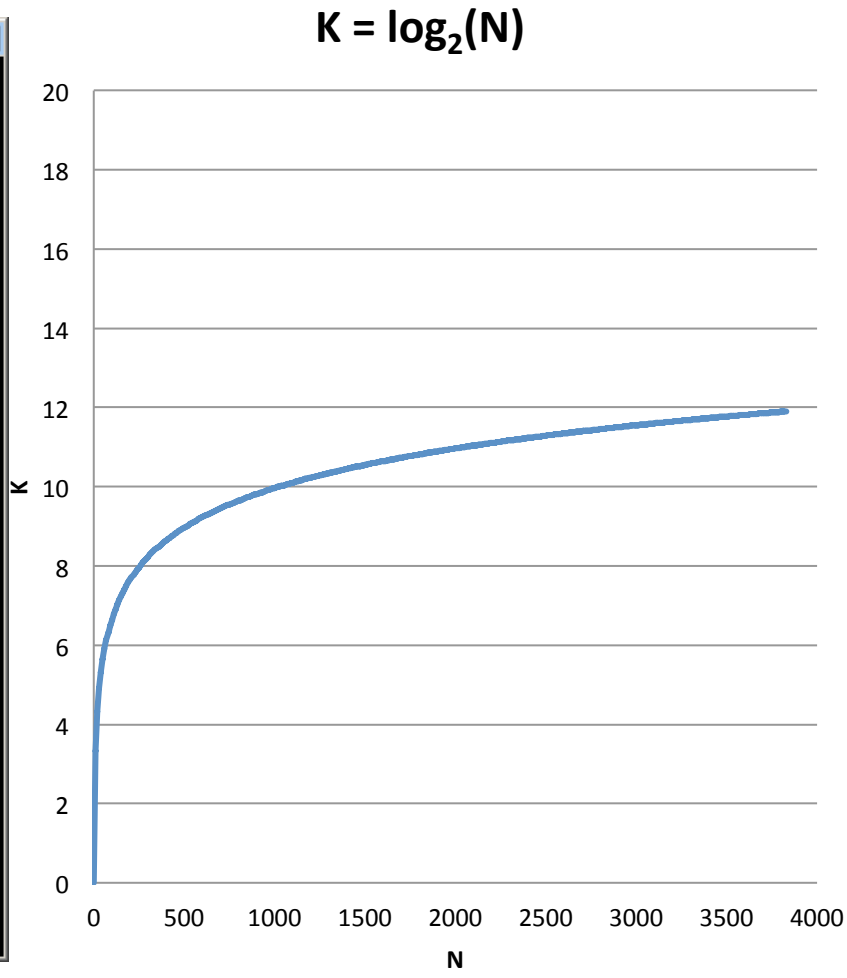
  Search stops when items to search ($N/2^K$) $\rightarrow$ 1

  i.e. $N = 2^K$, $\log_2(N) = K$

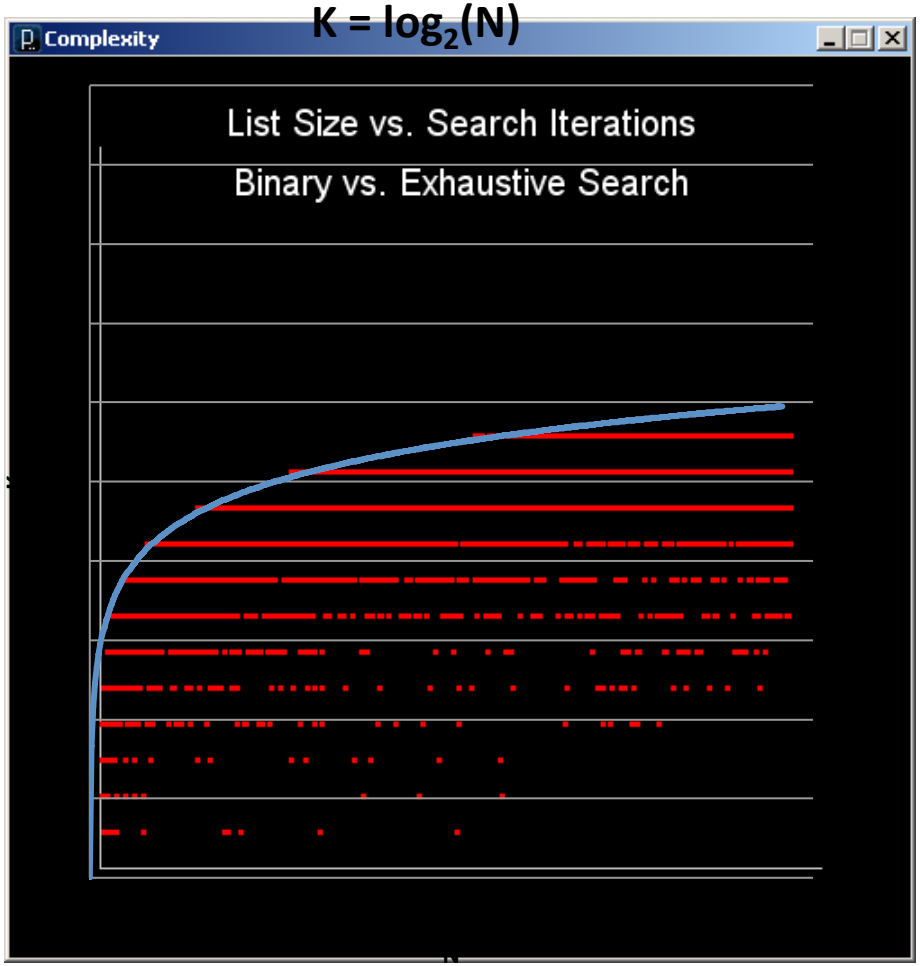
  Worst case: Number of iterations is $\log_2(N)$

It is said that Binary Search is a logarithmic algorithm and executes in O(logN) time.


Complexity
List Size vs. Search Iterations
Binary vs. Exhaustive Search


K = log2(N)
20
18
16
14
12
10
8
6
4
2
0
K
0
500
1000
1500
2000
2500
3000
3500
4000
N

$K = \log_2(N)$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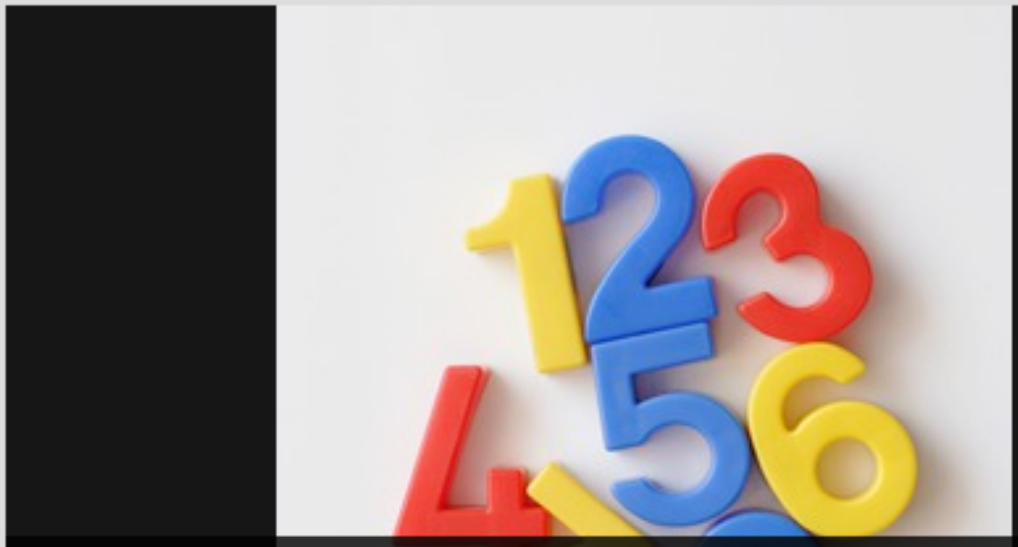
www.forbes.com/2011/06/16/weird-interview-questions_slide_11.html

Forbes.com

In Pictures: Weird Job Interview Questions



"Can I Guess?"

CLOSE

Given the numbers 1 to 1,000, what is the minimum number of guesses needed to find a specific number if you are given the hint "higher" or "lower" for each guess you make?

Asked at Facebook